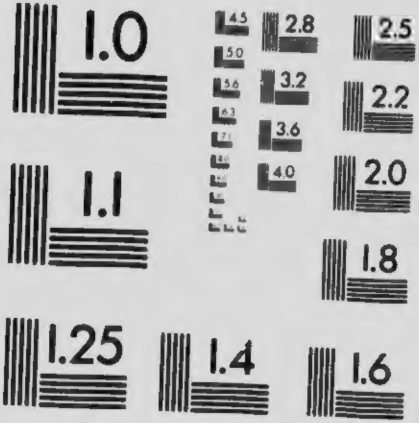
MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



APPLIED IMAGE Inc

1653 East Main Street
Rochester, New York 14609 USA
(716) 482 - 0300 - Phone
(716) 288 - 5989 - Fax

HANDBOOK

TO

ELEMENTARY ARITHMETIC

PARTS I AND II

70526

HANDBOOK

TO

ELEMENTARY ARITHMETIC

PARTS I AND II

FOR PUBLIC SCHOOLS

PRICE $1.00

W. J. GAGE & COMPANY, LIMITED
TORONTO

QA 106
H 35

Copyright, Canada, 1915, by W. J. GAGE & COMPANY, LIMITED
Toronto.

PREFACE

In presenting the answers to the Elementary Arithmetic, Parts I and II, together with hints for the solution of the more difficult problems, the authors desire to make the following observations:—

Great importance should be given to the oral arithmetic; in fact, no written work should be given to the pupil or attempted by the pupil until the principles necessary for the complete mastering of the subject have been thoroughly grasped through oral exercises involving the use of simple numbers. For this purpose the oral exercises of the text should be largely supplemented by the teacher. Emphasis should be laid upon actual business practices and the understanding of business terms. Often the difficulty in the solution of the problem disappears when these things are known.

Precision in the use of language necessary to the accurate expression of thought cannot receive too much attention either in oral or written work. This will promote not only keener discernment and mental development, but will also tend to secure accuracy in the final result. The habit of accepting inaccurate work upon the pretext that the *result does not matter* cannot be too strongly deprecated.

Second in importance to accuracy is *neatness* in the performance of all written work. Neatness will lessen the number of mechanical errors as well as lead to accuracy in reasoning. No written work in any subject will be accepted by the efficient teacher that does not closely approximate to the pupil's best production. To secure this will require vigilance and persistence on the part of the teacher, but the results will fully justify the effort put forth.

PREFACE

There are several thousand problems in the text from which a judicious selection should be made for a first course. This will leave new material for review. The ideal review in Arithmetic, any more than in any other subject, does not mean simply a *repetition* of what has been previously learned, but rather a *new* view of ground formerly traversed—either looked at from a different standpoint or freshened by the introduction of new material. There is but little vitality in solving the same old problems in the same old way.

All pupils in the same grade are not on the same level either in attainment or ability, and thus, if habits of indolence or mischief are not to be encouraged, exercises supplementary to the minimum required of all pupils should be provided for the more capable. Many problems of the text which are too difficult for the ordinary public school pupil will be valuable for this purpose, and if so used may tend to avert "arrested development" on the part of the rapid worker or the more brilliant in the class.

The necessity of keeping this book in small compass has in many cases caused a conciseness of statement beyond what is desirable in ordinary solution. A few errors were discovered in the first edition of the Arithmetic. These have been corrected in the second edition and the answers in the handbook are based on this edition. It is possible that in spite of the care exercised there may still appear some errors which have escaped notice. Information in regard to these will be gratefully received.

THE AUTHORS.

HANDBOOK

TO

ELEMENTARY ARITHMETIC

PARTS I AND II

FOR PUBLIC SCHOOLS

PART I

Exercise 7. Page 13

1. 16, 12, 18, 25, 28, 31, 30, 27. 2. 46, 39, 42, 48, 356, 409, 453.
3. 248, 314, 339, 316, 294, 318. 4. 68, 58, 50, 56, 577, 584, 652.
5. 68, 71, 75, 68, 760, 761, 673. 6. 88, 83, 80, 80, 701, 720, 788.
7. Sum of lines, 67, 65, 64, 68, 51, 58, 61, 52, 65, 66. Sum of columns, 64, 65, 58, 59, 69, 59, 58, 59, 64, 62. Sum [illegible] columns, 617. Sum of lines, 617.
8. 72 trees. 9. 12 qts. 10. 120 sheets.
11. $420. 12. $3.85. 13. 72 in.
14. 66 ft. 15. $1.62.

Exercise 8. Page 16

1. 25, 23, 15, 41, 22, 62, 43. 2. 84, 63, 70, 34, 23, 16, 52.
3. 29, 37, 46, 16, 34, 32, 51. 4. 19, 38, 56, 46, 38, 11, 56.
5. 30, 34, 45, 68. 6. 56, 29, 29, 19. 7. 19, 29, 46, 39.
8. 56, 9, 35, 37. 9. 44 and 12; 81 and 49; 111 and 37; 124 and 46; 127 and 29; 140 and 46; 161 and 9.
10. 104 and 46; 124 and 26; 119 and 15; 64 and 26; 96 and 54; 99 and 27; 73 and 35. 11. 18c.
12. 4 bouquets. 13. 6 ft.

Exercises 9-14 — Oral

PREFACE

THIS Handbook consists of three parts: Part I. gives the answers to the exercises, Part II. consists of solutions of the more difficult problems, and Part III. is a supplement giving additional information and discussing some topics which were omitted in the text-book.

The work has been prepared for the use of teachers and private students. With the multiplicity of duties ordinarily devolving on the teacher, time cannot always be had to solve all questions that may be presented by the pupil. Hence, a work such as this becomes a great convenience, if not an actual necessity.

It has not been thought best to solve such exercises as are comparatively easy or merely mechanical. Hence, many of those under Fractions, Extraction of Roots, Compound Numbers, Interest, etc., are omitted.

The solutions have, in general, been given with strict reference to the Unitary Method, thus showing its applicability to questions of every variety and degree of difficulty. They do not exhibit all the calculations at large, but they furnish results which serve to verify the operations at the successive stages of the process. In this way all that is necessary has been brought within a narrow compass, and the connection of the different parts of each solution will be more readily perceived.

It has not been the aim of the authors to make a mere Key, but to exhibit the best and neatest mode of working Arithmetical Exercises. Not only are neatness and method encouraged by the habit of arranging figures in their exact places, but the accuracy of the answer is best secured by the same means.

In the solutions of the problems in Chapter XVII., the actual business practice, so far as the authors can ascertain, has been followed.

Indications of any errors or obscurities will be thankfully received.

Exercise 15. Page 27

1. XIX, XXIV, XLIX, LXXXIV, XCIX.
2. CLXXXVII, CCVIII, DCCLXXXI, CMLXII, CMXCIX.
3. MCCCI, MCCCXC, MDCLXXXIV, MDCCCXV, MDCCCLXXVIII.
4. 44, 69, 94, 71.
5. 99, 129, 177.
6. 555, 1604, 1819, 1090.
7. 1290, 1009, 1490, 1900.
8. X, V, I.
9. C, L, V, I.

Exercise 16. Page 27

1. 100 times.
2. 9999. 1000.
3. 090, 900.
4. 26000, 10000, 89910, 399999, 40689, 20399.
5. 40000, 5000, 200, 60, 9; 3000000, 100000, 70000, 6000, 7. 7 hundreds and 4 tens; 16 hundreds and 8 tens; 100 hundreds.
6. 1914 = MCMXIV, 1915 = MCMXV, 1916 = MCMXVI, 1917 = MCMXVII, 1918 = MCMXVIII, 1919 = MCMXIX, 1920 = MCMXX, etc.
7. The answer for this question will vary from year to year. The most reliable figures will be obtained from the Dominion Census Bureau, Ottawa, Ontario.

Exercise 17. Page 30

1. 47 horses.
2. 98 boys.
3. 39 girls.
4. 978.
5. 956.
6. 898.
7. 889.
8. 879.
9. 979.
10. 697.
11. 798.
12. 998.
13. 898.
14. 879.
15. 889.
16. 8589.
17. 9879.
18. 8989.
19. 9989.
20. 98878.
21. 67988.
22. 88998.
23. 79988.

Exercise 18. Page 31

1. 79c.
2. 88 trees.
3. 589 ac.
4. $796.
5. 989 mi.
6. 969 yd.
7. 878 bales.
8. $8989.

Exercise 19. Page 33

1. 113 dollars.
2. 78 cents.
3. 152 boys.
4. 145 girls.
5. 146.
6. 247.
7. 162.
8. 161.
9. 217.
10. 213.
11. 255.
12. 205.
13. 150.
14. 418.
15. 306.
16. 255.
17. 2704.
18. 1656.
19. 1951.
20. 1976.
21. 1842.
22. 2141.
23. 23878.
24. 18204.
25. 16954.
26. 14978.
27. 15114.
28. 16046.
29. 36924.
30. 38487.
31. 41400.
32. 34826.
33. 30477.
34. 38548.
35. 276124.
36. 245389.
37. 185498.
38. 400123.

Exercise 20. Page 34

1. $535.14.
2. $7798.32.
3. $101990.

Exercise 21. Page 35

1. 24692.
2. 25879.
3. 27265.
4. 24447.
5. 23378.
6. 238390.
7. 246818.
8. 94903.
9. 76931.
10. 104974.
11. 47991.
12. 12521.
13. $4464.94.

Exercise 22. Page 35

Horizontal lines—$1099.28; $999.03; $1317.43; $966.84; $1015.23; $1128.17.

Vertical lines—$1731.98; $1426.17; $1294.68; $1172.96; $900.19. Total—$6525.98.

Exercise 23. Page 36

1. $222.
2. 1660 ac.
3. 120 da.
4. 1061 mi.
5. 936 lb.
6. 7428 bu.
7. 3441 ac.
8. $633.
9. 2104 pages.
10. $2237.

Exercise 24. Page 37

1. $31080.
2. 26728 mi.
3. $231,611,417.
4. 1407 mi.
5. 2065 mi.

Exercise 25. Page 40

1. 313. 2. 241. 3. 251. 4. 402.
5. 644. 6. 464. 7. 143. 8. 305.
9. 344. 10. 304. 11. 733. 12. 530.
13. 4442. 14. 5022. 15. 2223. 16. 2001.
17. 2530. 18. 4422. 19. 5512. 20. 2734.
21. 5024. 22. 6257. 23. 1361. 24. 4623.
25. 423. 26. 60224. 27. 36275 28. 31216.
29. 5082. 30. 43264. 31. 36425 32. 35137.
33. 66243. 34. 75331. 35. 61161. 36. 364.
37. 233. 38. 228. 39. 2533. 40. 1243.
41. 6216.

Exercise 26. Page 41

1. 43 girls. 2. 44 cents. 3. $16. 4. 34 runs.
5. 44. 6. $43. 7. $33. 8. $2112.

Exercise 27. Page 43

1. 4944. 2. 2857. 3. 5339. 4. 1299.
5. 1359. 6. 5247. 7. 2279. 8. 5263.
9. 3784. 10. 5682. 11. 4469. 12. 1789.
13. 11844. 14. 19528. 15 52888. 16. 35499.
17. 49289. 18. 25012. 19. 21236. 20. 61076.
21. 32130. 22. 78224 23. 54546. 24. 57754.
25. 43266. 26. 51837. 27. 47978. 28. 41094.
29. 32113. 30. 11523.

Exercise 28. Page 44

1. 642396, 576917. 2. 623141, 764544. 3. 475078, 1.
4. 709909, 616161. 5. 3428, 4249, 5144. 6. 3683, 1966, 2736.
7. 3998, 3466, 1613. 8. 1145, 5406, 2246. 9. $8.
10. 77 yd. 11. $560. 12. 1826.
13. 3251. 14. $3.44.

Exercise 29. Page 44

1. 520. 2. 6258.
3. 4216. 4. 16220.
5. 8240. 6. 11478 − 7110 = 4368.
7. 1971 + 127 = 2098. 8. 3936 + 507 = 4443.
9. 999001 − 1487 = 997514. 10. 5383 − 2992 = 2391.

Exercise 30. Page 45

1. 1328 − 1151 = 177. 2. 1621 − 882 = 739.
3. 3274 − 1463 = 1811. 4. 1744 − 1053 = 691.
5. 4382 − 2120 = 2262. 6. 1922 − 1402 = 520.
7. 5514 − 3335 = 2179. 8. 101021 − 78695 = 22326.
9. 93138 − 15273 = 77865. 10. 12210 − 8612 = 3598.

Exercise 31. Page 46

1. $357. 2. Loss $632.
3. 11438 − 904 = 10534.
4. First, $352; Second, $419; Third, $325; Fourth, $104.
5. 2325 − 1369 = 956. 6. 814 ft.

Exercise 32. Page 46

1. 78360 − 35951 = 42409. 2. 227 ac.
3. Red, 84; Black, 47; Blue, 76; Green, 38. Total 245.
4. Gladstone was 10 years older than Victoria when he died. He lived 7 years longer than Victoria.
5. (*a*) 39951, (*b*) 313863, (*c*) 353814.
6. 202750 − 168498 = 34252. 7. $17500 − $17461 = $39 gain.

Exercise 33. Page 47

1. 53c. 2. 984. 3. 2608c.
4. 17th from the bottom, 29th from the top.
5. 589 nuts. 6. $28. 7. 33c. gain.
8. $1269. 9. $1864. 10. $1.25.
11. 3 boys.

Exercise 34. Page 52

1. 14864
2. 16864.
3. 216936.
4. 368492.
5. 195 boys.
6. 282c.
7. 959 cows.
8. 1488 apples.
9. 2106 girls.
10. 1890.
11. 3360.
[illegible]. 3070.
13. 23526.
14. 47901.
15. 43710.
16. 78112.
17. 53838.
18. 70340.
19. 72028.
20. 661672.
21. 153132.
22. 630855.
23. 352794.
24. 646857.
25. 53936.
26. 54360.
27. 432481.

Exercise 35. Page 52

1. $162.80.
2. $1854.30.
3. $2709.
4. $15215.
5. 336 sheep ; $2352.
6. 94 lb.; $26.10; $7.52; $6.58.

Exercise 36. Page 54

1. 11950.
2. 40992.
3. 118377.
4. 66738.
5. 482544.
6. 340488.
7. 189945.
8. 240896.
9. 134010.
10. 62550.
11. 183576.
12. 1348560.
13. 98560 yd.
14. 68520 ft.
15. $83.52.
16. 459 da.
17. $3000.
18. $8505.
19. $19845.

Exercise 37. Page 56

1. 472440.
2. 300720.
3. 236196.
4. 562650.
5. 724885.
6. 6608822.
7. 6586169.
8. 6509916.
9. 1194872.
10. 4127874.
11. 9781440.
12. 11961586.
13. 14821755.
14. 25581580.
15. 23120856.
16. 81362385.
17. 29455710.
18. 31259060.
19. 70021632.
20. 14069499.
21. 41316048.
22. 26514000.
23. 42741832.
24. 16765686.
25. 66093951.
26. 217702278.
27. 163588743.
28. 307551216.
29. 276010344.
30. 114297351.
31. 348112465.
32. 283036032.

Exercise 38. Page 57

1. 127405. 2. 6317608. 3. 1960452. 4. 6825456.
5. 63366216. 6. 56127072. 7. 6749472. 8. 8214206.
9. 25996104. 10. 80071992. 11. 738110274. 12. 344892582.

Exercise 39. Page 57

1. 283800. 2. 592900. 3. 60744600. 4. 11887500.
5. 243000. 6. 258000. 7. 11214000. 8. 4096000.
9. 422500. 10. 627000000. 11. 64610000. 12. 488000000.

Exercise 40. Page 58

1. 454560 sheets. 2. 195559 yd. 3. $6125.
4. 1653 yd. 5. 3915648 lb. 6. $1228275.
7. 263952 apples. 8. 915750 pages. 9. 61275 yd.

Exercise 41. Page 58

1. $1246420. 2. $3926000. 3. $7080320.
4. 262800 bbl. 5. $277107850. 6. 372480 hills.
7. 89784 yd. 8. 7344 mi. 9. $631.80.
10. 1323000 lb.

Exercise 42. Page 59

1. 785664. 2. 693875 − 258420 = 435455.
3. 388692 + 113484 = 502176. 4. 389037 − 86271 = 302766.
5. 172081. 6. 46189.
7. 990991 × 1640 = 1625225240. 8. 502254 − 423654 = 78600.
9. 67419143. 10. 272 × 300 = 81600.

Exercise 43. Page 60

1. $2960 − $2352 = $608.
2. 164700 men − 1936 men = 162764 men.
3. 530229 gal. 4. $4773 − $2000 = $2773.

5. 36652c. – 24000c. = 12652c.
6. House, $7455, Farm, $9940.
7. Horses, $2532.
8. A's, 706800 letters; B's, 1126125 letters.

Exercises 44, 45 — Oral

Exercise 46. Page 66

1. 228.
2. 368.
3. 274.
4. 187.
5. 269.
6. 245.
7. 272.
8. 174.
9. 138.
10. 246.
11. 223.
12. 171.
13. 182.
14. 255.
15. 275.
16. 128.
17. 156.
18. 183.
19. 144.
20. 206.
21. 184.
22. 204.
23. 243.
24. 152.
25. 109.
26. 123.
27. 147.
28. 129.
29. 177.
30. 168.
31. 163.
32. 187.
33. 156.
34. 153.
35. 176.
36. 139.
37. 108.
38. 109.
39. 129.
40. 144.
41. 246.
42. 556.
43. 419.
44. 609.
45. 1223.
46. 367.
47. 676.
48. 1208.
49. 1337.
50. 1410.
51. 907.
52. 457.
53. 947.
54. 3669.
55. 13879.
56. 988.
57. 442.
58. 285.
59. 7032.
60. 7484.

Exercise 47. Page 68

1. 59 oranges.
2. 173 da.
3. 918 lb.
4. 231 yd.
5. 96 rd.
6. 91c.
7. 15 bu.
8. 123.
9. 6052 bu.

Exercise 48. Page 68

1. 432 bbl.
2. 1256 lb.
3. $6226.
4. $11.20.
5. $8.
6. 13$\frac{1}{3}$ apples.
7. 130$\frac{3}{4}$ lb.
8. $125.20.
9. 52$\frac{1}{7}$ wk.

Exercise 49. Page 71

1. 24 times and 20 remainder.
2. 8 “ 39 “
3. 18 “ 33 “
4. 74 [illegible] 53 “
5. 39 “ 50 “
6. 588 “ 51 “
7. 531 “ 45 “
8. 945 “ 27 “
9. 554 “ 66 “
10. 49 “ 49 “
11. 231 “ 231 “
12. 498 “ 153 “
13. 375 “ 375 “
14. 2259 “ 1 “
15. 5050 “ 99 “
16. 2831 “ 0
17. 23137 “ 136 “
18. 30653 “ 17 “
19. 6205 times and 162 remainder.
20. 6200 “ 160 “
21. 3746 “ 0 “
22. 2025 “ 372 “
23. 4998 “ 432 “
24. 9710 “ 140 “
25. 43210 “ 0 “
26. 4671 “ 0 “
27. 3180 “ 0 “
28. 3615 “ 0 “
29. 1142 “ 0 “
30. 7277 “ 9467 “
31. 25596 “ 11496 “
32. 50444 “ 0 “
33. 100300001 times and 0 remainder.
34. 10000260 times and 36 remainder.
35. 1003.

Exercise 50. Page 73

1. 1734 times and 9 remainder.
2. 1366 “ 44 “
3. 1549 “ 61 “
4. 307 “ 38 “
5. 246925 “ 21 “
6. 149147 “ 13 “
7. 349 times and 17 remainder.
8. 696 “ 26 “
9. 1082 “ 6 “
10. 4236 “ 57 “
11. 2570 “ 0 “
12. 5599 “ 89 “

Exercise 51. Page 73

1. 43 da.
2. 38 da.
3. 1090 ft.
4. $32.
5. 129 yr.
6. $123.
7. $46.
8. 545 lb.
9. 343$\frac{5}{7}$ mi.
10. 2075 bbl.

Exercise 52. Page 75

1 24 times and 5 remainder.
2. 127 " " 22 "
3. 12 " " 242 "
4. 326 " " 61 "
5. 804 " " 9 "
6. 1183 " " 38 "
7. 2 times and 386 remainder.
8. 5 " " 3500 "
9. 30 " " 9751 "
10. 153 " " 5521 "
11. 673 " " 1773 "
12. 632 " " 6279 "

Exercise 53. Page 75

1. 108 yd.
2. 65 hr.
3. 123 lb.
4. 30 lb.
5. 42 bu.
6. $235.
7. 237 bu.
8. 43 bu.

Exercise 54. Page 77

1. 4.
2. Dividend – (Divisor × Quotient).
3. 89.
4. (Divisor × Quotient) + Remainder.
5. 6.
6. (Dividend – Remainder) ÷ Divisor.
7. 12.
8. (Dividend – Remainder) ÷ Quotient.
9. 9265.
10. $(97 \times 203) + 96 = 19787$.

Exercise 55. Page 78

1. 271.
2. 555.
3. 222221.
4. 632819.
5. $(383 \times 587) - 313 = 224508$.
6. 37874.
7. 414348.
8. 16546.
9. 9576.

Exercise 56. Page 78

1. 38 mo. = 3 yr. 2 mo.
2. 367 ac.
3. $2310.
4. $3380.
5. 15 wk.
6. $44.
7. $65\frac{22}{25}$c.
8. 1650 bbl.
9. 24 mo. = 2 yr.

Exercise 57. Page 79

1. 80c.
2. $2.40.
3. 54c.
4. $56.
5. $96.
6. $100.
7. 40c.
8. $12.
9. $6880.

Exercise 58. Page 80

1. 9 da. 2. 15 da. 3. 12 da. 4. 32 da.
5. 361 da. 6. 50 da. 7. 90 da. 8. 48 da.

Exercise 59. Page 81

1. 28 men. 2. 105 men. 3. 21 men.
4. 108 men. 5. 25 men. 6. 114 men.
7. 56 men 8. 72 men. 9. 150 men.

Exercise 60 — Oral

Exercise 61. Page 86

1. 92*d.* 2. 1104 far. 3. £. 9. 15*s.* 5*d.*
4. £309 5*s.* 5. 2406*d.* 6. 560*d.*
7. £4. 1*s.* 5*d.* 2 far. 8. £29. 16*s.* 11*d.* 9. 183839 far
10. 3209 far. 11. $28.83½. 12. $18.43½.

Exercise 62. Page 88

1. 65296 oz. 2. 86 oz. 3. 13 cwt. 52 lb. 13 oz.
4. 30 lb. 10 oz. 6 dr. 5. 516608 dr. 6. 168005 oz.
7. 2 ton. 7 cwt. 74 lb. 1 oz. 8. 4933 oz.
9. 25 ton. 16 cwt. 49 lbs. 10. 26 stone. 1 lb.
11. 43136 dr. 12. 3 cwt. 90 lb. 10 oz.

Exercise 63. Page 90

1. 685 rd. 2. 3036 in. 3. 73 ft.
4. 70 in. = 5 ft. 10 in. 5. 217 yd. 2 ft. 9 in. 6. 14 mi. 82 rd.
7. 13464 in. 8. 9 ch. 46 ft. 9 in. 9. 5238 in

Exercise 64. Page 94

1. 10368 sq. in. 2. 13104 sq. in. 3. 124 sq. ft.
4. 28 sq. yd. 3 sq. ft. 127 sq. in. 5. 840 sq. rd.
6. 49 ac. 5 sq. rd. 7. 1120 sq. rd. 8. 300 sq. ch.
9. 87120 sq. ft. 10. 22129 sq. in. 11. 47 ac. 125 sq. rd.
12. 51200 sq. rd

Exercise 65. Page 95

1. Square; 160 ac.
2. 480 ac.; 4 mi. of fence; cost $384.
3. $160.
4. $5520.
5. 1 mi.
6. $334.30.
7. $1604160.
8. 2 mi.; 4 mi.
9. (1) 3 mi. 66 ft. or 3 mi. 4 rd.
   (2) 7 mi. 66 yd. = 7 mi. 12 rd.
   (3) The distance to the centre of the road allowance midway between the north and the south sides of sections 23 and 24 = 6½ mi. 99 yd. = 6 mi. 979 yd. = 6 mi. 178 rd.
   (4) The distance will be 6½ mi. 99 yd. = 6 mi. 178 rd.

Exercise 66. Page 98

1. 60 cords 9 cu. ft.
2. 4 cu. ft. 1557 cu. in.
3. 435424 cu. in.
4. 610 cord ft. 4 cu.ft.
5. 813888 cu. in.
6. 3552 cu. ft.
7. 13 cu. yd. 18 cu. ft. 52 cu. in.
8. 45 cords 8 cu.ft.
9. 3712 cu. ft. 260 cu. in.
10. 186624 cu. in.

Exercise 67. Page 100

1. 120 bu. 2 qt.
2. 678 pt.
3. 2311 pt.
4. 336 gal. 3 qt. 1 pt.
5. 556 pk.
6. 61 bu. 25 lb.
7. 96 gal. 8 lb. or $96\frac{8}{10}$ gal.
8. 80 standard gal.
9. 157 bu. 10 lb.
10. 72 bu.

Exercise 68. Page 101

1. 62100 sec.
2. 230400 sec.
3. 10 hr. 14 min. 1 sec.
4. 2 da. 13 hr. 18 min.
5. 1814400 sec.
6. 8784 hr.
7. 2 wk. 3 da. 17 hr.
8. 38 da. 13 hr. 55 min.
9. 608400 sec.
10. 1 da. 22 hr. 47 min. 36 sec.

Exercise 69. Page 102

1. 1740 sec.
2. 34099 sec.
3. 12° 48′.
4. 21° 21′ 35″.
5. 19038 sec.
6. 2714400 sec.
7. 1299660 sec.
8. 1641648 sec.

Exercise 70. Page 103

1. 29 bbl. 84 lb.
2. 1608 sheets.
3. 5292 units.
4. 34 bbl. 47 lb.
5. 125 doz.
6. 69 score.
7. 4 great gross, 5 gross, 4 doz., 4 units.
8. 683 lb.
9. 1148 lb.
10. 16 reams 7 quires 21 sheets.

Exercise 71. Page 103

1. 672 hr.
2. 2640 yd.
3. 317520 sq. in.
4. 1168 qt.
5. 848 cu. ft.
6. 33 wk. 3 da. 4 hr.
7. 33481 far.
8. 439552 dr.
9. 2855520 in.
10. 172200 sec.
11. 240 lb. heavier.

Exercise 72. Page 104

1. 26112 oz.
2. $4.08.
3. $4.05.
4. $3045.
5. 39 mi. 510 yd.
6. 2678400 sec.
7. 8600 yd.
8. $13.65.
9. 92 bu. 28 lb.
10. 141 bu. 16 lb.
11. 23 tons 5 cwt. 62 lb. 8 oz
12. 1666 sq. yd. 6 sq. ft.
13. $595.
14. 17190 ft.
15. $195.

Exercise 73. Page 105

1. 73 cwt. 70 lb. 9 oz.
2. 16 yd. 1 ft. 6 in.
3. £34. 14*s.* 8*d.*
4. 43 bu. 1 pk. 1 pt.
5. 16 yd. 2 ft. 3 in.
6. 6 wk. 3 da. 6 hr. 50 min. 33 sec.
7. 13 yd. 8 in.
8. 101 mi. 1261 yd.
9. 1395 cords 124 cu. ft.
10. 458 bu. 49 lb.
11. 99 ac. 4488 sq. yd. 6 sq. ft.

Exercise 74. Page 107

1. 19 mi. 1758 yd. 2 ft.
2. 58 ac. 4834 sq. yd. 5 sq. ft.
3. 5 mi. 1759 yd. 1 ft. 8 in.
4. £27. 17*s.* 4*d.*
5. 13 oz. 11 dr.
6. 142 bu. 2 pk. 1 gal. 1 qt.
7. 31 gal. 2 qt. 1 pt.
8. 12 sq. yd. 3 sq. ft. 132 sq. in.
9. 1 cwt. 85 lb.
10. 72 mi. 254 rd.

Exercise 75. Page 108

1. 4 ac. 4054 sq. yd. 4 sq. ft.
2. 1 hr. 35 min. 45 sec.
3. 62° 49′ 4″.
4. 9 ac. 3534 sq. yd.
5. 1 hr. 41 min. = 6060 sec.
6. 1 bu. 1 pk. 1 gal.
7. 4 tons 1323 lb.
8. 8 yd. 2 ft. 3 in.
9. 114 cords 11 cu. ft.
10. 500 gal.
11. 97 [illegible] 842 yd. 2 ft. 6 in.
12. 865 cords 6 cord ft.

Exercise 76. Page 111

1. 90 cwt. 82 lb. 13 oz.
2. 75 da. 23 hr. 34 min. 40 sec.
3. £600 9s. $6\frac{3}{4}$d.
4. 249 tons 5 cwt. 25 lb.
5. 150 ac. 115 sq. rd.
6. 23058 gal.
7. 88 mi. 120 rd.
8. £54 18s. $0\frac{1}{2}$d.
9. 662 mi. 1024 yd.

Exercise 77. Page 111

1. 2739 bu. 1 pk. 1 gal. 1 qt.
2. 55 min. 15 sec.
3. 402 cords 92 cu. ft.
4. 17 min. 58 sec.
5. 2215 mi. 1228 yd.
6. 12 tons 12 cwt. 70 lb.
7. 54 bu.
8. 87 mi.
9. 455 bu.
10. 1532 tons 16 cwt. 25 lb.
11. 468 yd. 8 in.
12. 23 tons 3 cwt. 82 lb.
13. 12 mi. 1466 yd. 2 ft.
14. 6 ac. 1960 sq. yd.

Exercise 78. Page 114

1. £15 9s. 7d.
2. 16 tons 2 cwt. 38 lb.
3. 1 gal. 2 qt.
4. 2 cu. yd. 6 cu. ft. 960 cu. in.
5. 9 mi. 1322 yd. 11 in.
6. 25 wk. 3 da. 18 hr. 55 min. 18 sec.
7. 13 lb. 11 oz. 6 dwt. 15 gr. and 35 gr. remainder.
8. 27 cu. yd. 4 cu. ft. 246 cu. in. and 34 cu. in. remainder.
9. 7 tons 1614 lb. 5 oz.

Exercise 79. Page 115

1. 8. 2. 10 times and 18 qt. remainder
3. 11 times and 480 oz. remainder (avoirdupois).
4. 5 times and 355 qt. remainder.
5. 2 bu. 3 pk. $3\frac{3}{4}$ qt. 6. 25 jars.
7. 5 wk. 8. 17 coats and 2 yd. 9 in. left.
9. 42240 times.

Exercise 80. Page 115

1. 37 vessels. 2. $10249\frac{7}{17}$ times. 3. 4 mi. 328 yd.
4. $28\frac{4}{5}$ seconds. 5. 166 hr. 40 min. 6. 8 hr.
7. 42 doz. and 3. 8. 216 tons. 2 cwt. 98 lb.
9. 27 mi. 480 yd. 10. \$96.
11. Wheat, \$48.24; timothy seed, \$48.75; rye, \$18.72; oats, \$25.55. Total, \$141.26.

Exercise 81. Page 121. See Model Receipt.

Exercise 82. Page 121

1. \$32.20. 2. \$13.76. 3. \$3.85.
4. \$27.76. 5. \$47.02.
6. Wheat, \$187.61; oats, \$171.00; corn, \$34.20; peas, \$143.50; barley, \$253.23. Total, \$789.54.
7. \$14.24.
8. Boots, \$121.50, gaiters, \$312; overshoes, \$114.66; slippers, \$17.10; heavy boots, \$206.25. Total, \$771.51.
9. \$18.78 10. \$25.41 – \$22.10 = \$3.31.
11. \$44.25 – \$34.07 = \$10.18.

Exercise 83. Page 123

1. $34\frac{1}{4}$. 2. $32\frac{4}{9}$. 3. $493\frac{1}{3}$.
4. $54\frac{2}{5}$. 5. 44. 6. 8 lb. 7 oz.
7. 110; 10. 8. \$457.
9. Average per load, 55 bu. 24 lb.; average per acre, 22 bu. $9\frac{3}{8}$ lb.
10. 456; 465; 564; 546; 651; 615. Total, 3330. Average, 555.

Exercise 84. Page 124

1. 65 lb. 3 oz. 2. 135 mi. 215 rd. 3. $67\frac{6}{7}$c.
4. $2.06¼. 5. Lost 30c. 6. 40c.
7. Wheat worth 78c. sold at 76c.; loss = 2c. per bushel.
" " 72c. " 76c.; gain = 4c. "
To neither gain nor lose he must sell 4 bu. at 78c. to 2 bu. at 72c., or 200 bu. at 78c. to 100 bu. at 72c.
8. 25¼ lb. 9. $10.20.
10. 482 pages. 11. 27 mi. 547 yd.

Exercise 85. Page 126

1. $3.50 and $3.75. 2. 52 and 61. 3. 268 and 375.
4. 57 lb. 5 oz. and 64 lb. 12 oz.
5. 112 ac. 2452½ sq. yd. and 113 ac. 2684½ sq. yd.
6. A, 45c.; B, 83c.; C, 79c.
7. A, 16 marbles; B, 23 marbles; C, 33 marbles.
8. First, 540 pages; second, 284 pages; third, 176 pages.
9. 478 and 521.
10. Length, 144 yd.; width, 117 yd.

Exercise 86. Page 127

1. 30 and 120. 2. A, $120; B, $80. 3. $700 and $560.
4. A, $125; B, $150; C, $225. 5. A, 7c.; B, 14c.; C, 42c.
6. A, 25c.; B, 90c. 7. A, $25; B, $54; C, $78.
8. There will be 2 five-cent pieces and 3 ten-cent pieces for every 1 twenty-five-cent piece; i. e., for one division 10c. + 30c. + 25c. = 65c. will be required.
No. of divisions = $3.90 ÷ 65c. = 6.
" five-cent pieces = 2 × 6 = 12; value 60c.
" ten-cent " = 3 × 6 = 18; " $1.80.
" twenty-five-cent pieces = 1 × 6 = 6; value, $1.50.
9. Wheat 54 bu.; barley, 135 bu.
10. Green, 138; blue, 112; red, 100.
11. A, 30 cords 60 cu. ft.; B, 40 cords 35 cu. ft.; C, 41 cords 33 cu. ft.
12. A, $50.75; B, $70.25; C, $80.65; D, $90.25.

Exercise 87. Page 128 — Oral

Exercise 88. Page 128

1. 96 sq. ft.
2. 91 sq. ft.
3. 525 sq. ft.
4. 18 sq. yd. 6 sq. ft.
5. 351 sq. yd. 3 sq. ft.
6. 470 sq. ft.
7. Walls and ceiling, $14.43⅓.
8. 4 ac. 20 sq. rd.
9. 1404 bricks.
10. 5 sq. ft. 142½ sq. in.

Exercise 89. Page 129

1. 10 ft. 6 in.
2. 3 ft.
3. 2 ft. 8 in.
4. 71 yd. 2 ft. 4 in.
5. 16 ft.
6. 15 ft.
7. 12 ft.
8. 15 in.
9. 63 ft.

Exercise 90. Page 130

1. (i) If carpet run lengthwise, No. strips = 10 ; No. yd. = 90.
    " " crosswise " " = 12 ; " = 84.
   (ii) " " lengthwise " " = 6 ; " = 30.
    " " crosswise " " = 7 ; " = 28.
   (iii) " " lengthwise " " = 8 ; " = 64.
    " " crosswise " " = 11 ; " = 66.
   (iv) " " lengthwise " " = 12 ; " = 144.
    " " crosswise " " = 16 ; " = 138⅔.
4. Carpet running lengthwise, No. yd. = 40 ; cost = $48.
5. " " " " = 64 ; " = $57.60.
6. " " " " = 29⅙ ; " = $29.16⅔.
7. (i) 5 strips ; (ii) 7 strips.
8. 8 strips.
9. 8 strips.
10. $42.75.
11. Carpet running lengthwise, No. strips = 8.

    No. yds. of carpet $= \frac{57.60}{1.20} = 48$. Length of each strip = 6 yd.

    Length of room = 18 ft.
12. $9.47⅔.

Exercise 91. Page 132

1. 32 sq. yd.
2. 150 sq. yd. 6 sq. ft.
3. $37.80.
4. 12 ft. $6\frac{6}{11}$ in.
5. Walls, 824 sq. ft.; walls and ceiling, 1292 sq. ft.
6. $44.
7. $5.
8. 240 ft.
9. $29.40.

Exercise 92. Page 133

1. 47 strips = 141 yd.
2. Walls and ceiling, 829 sq. ft. 135 sq. in.
3. 112 yd.
4. $14.
5. 348 yd.; 44 single rolls.
6. 107 strips; 81 rolls; cost, $20.25.
7. Double rolls, $1.20; single rolls, $1.50.
8. $147\frac{17}{21}$ yd.

Exercise 93. Page 135

1. 10 board ft.
2. $13\frac{1}{2}$ board ft.
3. 40 board ft.
4. 24 board ft.
5. 624 board ft.
6. 2400 board ft.
7. 2232 board ft.
8. 25920 board ft.
9. 3000 board ft.
10. $720.
11. $160.
12. $396.

Exercise 94. Page 136

1. 2592 shingles.
2. 3888 shingles.
3. 5760 shingles.
4. 9600 shingles.
5. 18 M.
6. 20 ft. 10 in.
7. 12 ft. 6 in.
8. $105.30.
9. $40.
10. $28.80.

Exercise 95. Page 137

1. 240 cu. ft.
2. 93 cu. ft. 576 cu. in.
3. 187 cu. ft. 864 cu. in.
4. 35 cu. ft.
5. 20250 bricks.
6. 116.14\frac{22}{27}$ or $116.15.
7. 9 ft.
8. $4\frac{4}{5}$ ft.
9. $31\frac{1}{4}$ cords.
10. 166 cu. ft. 432 cu. in.

Exercise 96. Page 138

1. 18 sheep. 2. 70c. 3. $5.15.
4. 100 shares. 5. 16 times. 6. $3.75.
7. $122. 8. $93.84. 9. 50 tons.
10. $1.10. 11. $1191.75. 12. 1760.
13. 476 yd.; 30c. gain per yd. 14. 400 bu.
15. 1560 pairs.

Exercise 97. Page 142

1. 880, 4144, 1000 and 1296 are divisible by 2, 4 and 8;
1356 and 4044 are divisible by 2 and 4;
5214 and 125474 are divisible by 2.
2. 5125, 4300, 41250 and 3225 are divisible by 5.
3. 1236, 4344, 203640 are divisible by 3 and 6.
1239 is divisible by 3.

Exercise 98. Page 143

1. 2, 2, 2, 2, 3. 2. 2, 2, 2, 3, 3. 3. 3, 3, 3, 3.
4. 2, 2, 3, 3, 3. 5. 5, 5, 7. 6. 2, 2, 2, 2, 2, 5.
7. 5, 5, 13. 8. 3, 11, 13. 9. 2, 2, 3, 23.
10. 2, 2, 2, 2, 2, 5, 5. 11. 3, 3, 5, 19. 12. 3, 5, 7, 11.
13. 5, 5, 5, 5, 2. 14. 2, 2, 2, 2, 2, 2, 2, 2, 2, 2.
15. 5, 5, 5, 11.

Exercise 99. Page 143

1. 2, 5. 2. 3. 3. 2. 4. 3, 3.
5. 2, 2, 2. 6. 3, 7. 7. 2, 2, 2, 3, 5.
8. 3. 5, 7. 9. 2, 2, 5, 5. 10. 2, 2, 2, 3, 5.

Exercise 100. Page 143

1. 7, 14, 21, 28, 35, 42, 49.
2. 208, 221, 234, 247, 260, 273, 286, 299.
3. 5, 10, 15, 20, 25, 30, 35, 40. 4. 5 or 0.
5. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99. 108, 117, 126.
6. 0. 7. 21, 23, 25, 27, 29, 31, 33, 35, 37, 39.
8. 11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132, 143.
9. 9, 21, 243, 193, 83.

Exercise 101. Page 145

1. 2. 2. 12. 3. 12. 4. 18.
5. 30. 6. 72. 7. 20 yd. 8. 72 bu.
9. $22. 10. 135 yd. 11. $2.

Exercise 102. Page 147

1. 6. 2. 12. 3. 16. 4. 13.
5. 21. 6. 5. 7. 4. 8. 8.
9. 14. 10. 10. 11. 42. 12. 24.
13. 11. 14. 75.

Exercise 103. Page 149

1. 23. 2. 37. 3. 41. 4. 56
5. 45. 6. 61. 7. 42. 8. 11.
9. 1. 10. 21.

Exercise 104. Page 149

1. 8 ft. 2. 21 ft. 3. 16 ft.
4. 4 qt. 5. 33 pupils in each class.
6. 19 lb. in each parcel. 7. 21.
8. 27 mi. 9. 32 gal. 10. 220 yd.
11. 6 ac.; 16 fields. 12. 8 oz.

Exercise 105. Page 152

1. $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$. 2. $2 \times 2 \times 2 \times 3 \times 3 = 72$.
3. $2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216$. 4. $2 \times 2 \times 3 \times 5 \times 7 = 420$.
5. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 = 448$.
6. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 20160$.

Exercise 106. Page 152

1. 30. 2. 60. 3. 36. 4. 150. 5. 300.
6. 180. 7. 360. 8. 770. 9. 2520. 10. 1512.

Exercise 107. Page 153

1. 173. 2. 2100 3. 360 bu. 4. 240c.
5. 84 bu. 6. 105 gal. 7. 546 ac. 8. 882 gal.

Exercise 108. Page 154

1. 9c. 2. 356.
3. Length of rail = 13 ft; No. of rails = 9672.
4. 75c. 5. $1080.
6. 3600. 7. 10,565,999.
8. 1,267,994,828,100. 9. Book work; 10296.
10. 4 and 3.

Exercise 109. Page 160

1. $7\frac{1}{2}$, $9\frac{1}{2}$, $8\frac{1}{2}$, $10\frac{1}{2}$, $12\frac{1}{2}$, $11\frac{1}{2}$. 2. $6\frac{1}{3}$, $7\frac{2}{3}$, $9\frac{1}{3}$, $11\frac{2}{3}$, $12\frac{2}{3}$, $10\frac{1}{3}$.
3. $4\frac{1}{4}$, $5\frac{1}{4}$, $6\frac{3}{4}$, $8\frac{1}{4}$, $8\frac{3}{4}$, $12\frac{1}{4}$. 4. $7\frac{3}{5}$, $8\frac{3}{5}$, $11\frac{1}{5}$, $12\frac{4}{5}$, $11\frac{4}{5}$, $12\frac{2}{5}$.
5. $12\frac{1}{3}$, $10\frac{5}{6}$, $11\frac{1}{3}$, $8\frac{5}{6}$, $11\frac{5}{6}$, $11\frac{1}{3}$. 6. $12\frac{3}{8}$, $11\frac{3}{8}$, $10\frac{7}{8}$, $9\frac{3}{8}$, $7\frac{7}{8}$, $6\frac{1}{2}$.
7. $8\frac{1}{10}$, $10\frac{1}{5}$, $7\frac{2}{5}$, $6\frac{9}{10}$, $8\frac{2}{5}$, $5\frac{3}{10}$. 8. $6\frac{9}{13}$, $4\frac{1}{13}$ ft., $8\frac{5}{13}$ in., $3\frac{9}{13}$ lb.

Exercise 110. Page 164

1. $\frac{9}{2}$. 2. $\frac{21}{4}$ 3. $\frac{38}{5}$. 4. $\frac{47}{6}$. 5. $\frac{41}{9}$. 6. $\frac{37}{5}$.
7. $\frac{83}{10}$. 8. $\frac{67}{12}$. 9. $\frac{84}{5}$. 10. $\frac{195}{8}$. 11. $\frac{130}{7}$. 12. $\frac{247}{12}$.

Exercises 111, 112 — Oral

Exercise 113. Page 169

1. $\frac{9}{12}$, $\frac{5}{12}$. 2. $\frac{12}{15}$, $\frac{7}{15}$. 3. $\frac{14}{16}$, $\frac{5}{16}$. 4. $\frac{8}{12}$, $\frac{9}{12}$.
5. $\frac{8}{20}$, $\frac{15}{20}$. 6. $\frac{9}{12}$, $\frac{10}{12}$. 7. $\frac{21}{24}$, $\frac{10}{24}$. 8. $\frac{14}{20}$, $\frac{15}{20}$.
9. $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$. 10. $\frac{20}{30}$, $\frac{25}{30}$, $\frac{18}{30}$. 11. $\frac{12}{24}$, $\frac{21}{24}$, $\frac{20}{24}$. 12. $\frac{30}{36}$, $\frac{16}{36}$, $\frac{21}{36}$.
13. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$; $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$; $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$. 14. $\frac{5}{8}$, $\frac{3}{4}$, $\frac{2}{3}$; $\frac{1}{2}$, $\frac{4}{9}$, $\frac{1}{6}$; $\frac{5}{6}$, $\frac{3}{4}$, $\frac{5}{9}$
15. Greatest $\frac{4}{5}$, least $\frac{2}{3}$; greatest $\frac{5}{6}$, least $\frac{3}{4}$; greatest $\frac{5}{6}$, least $\frac{7}{12}$.
16. $\frac{7}{9}$, $\frac{3}{4}$. 17. $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$. $\frac{40}{60}$, $\frac{24}{60}$, [illegible].

Exercise 114. Page 170

1. $\frac{11}{12}$. 2. $1\frac{1}{3}$. 3. $1\frac{3}{20}$. 4. $1\frac{7}{12}$.
5. $1\frac{1}{12}$. 6. $1\frac{11}{12}$. 7. $1\frac{11}{12}$. 8. $1\frac{2}{3}$.
9. $2\frac{1}{12}$. 10. $1\frac{5}{18}$. 11. $1\frac{31}{60}$. 12. $7\frac{1}{12}$.
13. $10\frac{5}{12}$. 14. 15. 15. $15\frac{1}{3}$. 16. $13\frac{7}{24}$.

Exercise 115. Page 171

1. $5\frac{22}{27}$.
2. $\frac{14}{15}$.
3. $82\frac{1}{4}$.
4. $4\frac{1}{6}$ tons.
5. $42\frac{7}{80}$ mi.
6. $10\frac{31}{60}$ yd.
7. $8\frac{7}{24}$ yd.
8. $29\frac{1}{16}$.
9. $141\frac{1}{4}$ rd.
10. $56\frac{29}{64}$ lb.

Exercise 116. Page 172

1. $\frac{1}{6}$
2. $\frac{1}{12}$
3. $\frac{4}{15}$.
4. $\frac{1}{30}$.
5. $\frac{3}{20}$.
6. $\frac{1}{4}$.
7. $\frac{1}{8}$.
8. $\frac{1}{24}$.
9. $\frac{1}{9}$.
10. $\frac{1}{36}$.
11. $\frac{7}{45}$.
12. $\frac{1}{24}$.

Exercise 117. Page 172

1. $\frac{17}{24}$.
2. $\$9\frac{5}{8}$; $\$22$.
3. $37\frac{1}{12}$ bu.
4. $10\frac{5}{9}$ da.
5. $11\frac{7}{8}$ rd.
6. $2\frac{5}{8}$.
7. $33\frac{5}{8}$ mi.

Exercise 118. Page 174

1. $3\frac{1}{2}$.
2. $1\frac{3}{8}$.
3. $4\frac{1}{2}$.
4. 9.
5. $11\frac{1}{4}$.
6. $22\frac{2}{5}$.
7. $27\frac{1}{2}$.
8. $22\frac{1}{2}$.
9. 51.
10. $\$1.87\frac{1}{2}$.
11. $\$30$.
12. $\$13.14$.

Exercise 119. Page 175

1. $\frac{5}{16}$.
2. $\frac{3}{13}$.
3. $\frac{3}{10}$.
4. $1\frac{1}{3}$.
5. $2\frac{1}{8}$.
6. $3\frac{2}{7}$.
7. $3\frac{1}{6}$.
8. $2\frac{1}{20}$.
9. $3\frac{8}{25}$.
10. $3\frac{11}{16}$.
11. $3\frac{1}{4}$ ac.
12. $2\frac{1}{4}$ cords.

Exercise 120 – Oral

Exercise 121. Page 177

1. $\frac{5}{12}$.
2. $\frac{35}{72}$.
3. $\frac{12}{133}$.
4. $\frac{13}{28}$.
5. $\frac{45}{88}$.
6. $\$1$.
7. $\$3$.
8. $\frac{7}{15}$.
9. $\frac{7}{20}$.
10. $\frac{4}{7}$.
11. $\frac{5}{14}$.
12. $\frac{5}{8}$.

Exercise 122. Page 177

1. $10\frac{1}{2}$.
2. $3\frac{17}{20}$.
3. $7\frac{7}{8}$.
4. $17\frac{1}{2}$.
5. $7\frac{7}{8}$.
6. $8\frac{4}{5}$.
7. $10\frac{1}{3}$.
8. $4\frac{2}{3}$.
9. $10\frac{2}{15}$.

Exercise 123. Page 178

1. 15. 2. 40. 3. $35\frac{5}{7}$. 4. $54\frac{3}{8}$.
5. $\frac{16}{25}$. 6. $\frac{3}{50}$. 7. $\frac{3}{8}$. 8. $\frac{7}{44}$.
9. 1. 10. $\frac{10}{63}$. 11. $\frac{7}{18}$. 12. $\frac{1040}{1281}$.

Exercise 124. Page 179

1. 15. 2. 16. 3. 21. 4. 2.
5. $\frac{1}{6}$. 6. $\frac{4}{5}$. 7. $\frac{3}{4}$. 8. $\frac{13}{18}$.
9. $\frac{2}{3}$.

Exercise 125. Page 179

1. 32c. 2. 24 bu. 3. $139\frac{1}{16}$. 4. $22\frac{2}{9}$ bu.
5. $2\frac{40}{63}$ wk. 6. 11 persons. 7. $3\frac{3}{8}$ wk. 8. $11\frac{1}{4}$ bu.

Exercise 126. Page 180

1. $\frac{69}{10}$. 2. $\frac{56}{5}$. 3. $\frac{18}{2}$. 4. $\frac{52}{4}$. 5. $\frac{80}{5}$. 6. $\frac{66}{1}$.
7. $\frac{11}{3}$. 8. $\frac{57}{8}$. 9. $\frac{79}{16}$. 10. $\frac{31}{8}$. 11. $\frac{60}{7}$. 12. $\frac{101}{14}$.
13. 11. 14. 6. 15. $17\frac{3}{5}$. 16. 8. 17. $26\frac{11}{16}$. 18. $12\frac{1}{2}$.
19. $\frac{4}{11}$. 20. $\frac{16}{35}$. 21. $\frac{13}{15}$. 22. $\frac{1}{2}$. 23. $\frac{5}{7}$. 24. $\frac{9}{10}$.
25. $\frac{24}{36}$, $\frac{28}{36}$, $\frac{27}{36}$. 26. $\frac{36}{126}$, $\frac{112}{126}$, $\frac{27}{126}$.
27. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$.

Exercise 127. Page 180

1. $1\frac{5}{12}$. 2. $1\frac{3}{10}$. 3. $\frac{41}{63}$. 4. $1\frac{101}{120}$. 5. $19\frac{1}{2}$.
6. $7\frac{5}{28}$. 7. $298\frac{1}{16}$. 8. $811\frac{3}{14}$. 9. $\frac{16}{33}$. 10. $4\frac{31}{60}$
11. $\frac{25}{42}$. 12. $\frac{29}{35}$. 13. $4\frac{3}{16}$. 14. $5\frac{5}{48}$. 15. $21\frac{3}{4}$.

Exercise 128. Page 181

1. $\frac{3}{5}$. 2. $1\frac{1}{8}$. 3. $1\frac{1}{2}$. 4. 1. 5. $3\frac{3}{5}$. 6. $\frac{4}{7}$.
7. $9\frac{9}{11}$. 8. $4\frac{17}{22}$. 9. $3\frac{6}{7}$. 10. 27. 11. $7\frac{1}{8}$. 12. $4\frac{18}{25}$.
13. $1\frac{1}{2}$. 14. $2\frac{2}{35}$. 15. $\frac{1}{36}$. 16. 42. 17. $11\frac{2}{3}$. 18. $7\frac{1}{4}$.
19. $\$5.00 - \$1.71\frac{5}{8}$ (i. e., $1.72) = $3.28.
20. $2.75. 21. 90c. 22. $16.80.
23. $\$34\frac{3}{4} = \$34.75$. 24. $36\frac{3}{8}$ in. by $22\frac{1}{2}$ in.

Exercises 129, 130 — Oral

Exercise 131. Page 185

1. 65.046.
2. 600.7354.
3. 4475.105045.
4. 2.4397464.
5. 101.209.
6. 10.867.
7. 114.1377.
8. 959.0483.
9. 40.52753.
10. 114.646.

Exercise 132. Page 186

1. 5.18. 2. 26.78. 3. 5.78. 4. 6.35. 5. 1.224. 6. 4.544.
7. 4.55 and 3.365.
8. 3.55, 15.092, and 1.125.
9. 7.357.
10. 4.295.

Exercise 133. Page 186

1. 7.485. 2. 44.235. 3. 7.942. 4. 56.571 ac. 5. 17.4.
6. 11.291 ; 6.875 ; 19.78.
7. This question is somewhat ambiguous. If he sold $\frac{875}{1000}$ of what he owned, then he would have left $\frac{125}{1000}$ of $\frac{9}{10}$ of the farm $= \frac{1125}{10000}$ farm $= .1125$ farm.
If he sold $\frac{875}{1000}$ of the farm, then he would have left $(.9 - .875)$ farm $= .025$ of the farm.
8. 9.71.
9. 1.086.
10. 62.225.

Exercise 134. Page 187

1. .988.
2. Wheat .48 of farm = 48 ac.
   Oats .14 " = 14 ac.
   Barley .065 " = 6.5 ac.
   Uncleared .315 of farm = 31.5 ac.
3. 3.942.
4. 1.863.
5. The third number should read one hundred, and one-hundredth. The result then would be 2102.115. The sum of the numbers as they stand = 2003.115.
6. 1.65 mi. west.
7. Perimeter = 63.294 in.; length is 4.483 in. greater than the width.
8. 54.175 ft.

Exercise 135. Page 188

1. 375. 2. .56. 3. 139.84. 4. 37.5.
5. 3.43. 6. 768.55. 7. 3.75. 8. 56.4.
9. 275.04. 10. Book work. 11. $241.345. 12. $32.884.

Exercise 136. Page 189

1. $320, $63, $108. 2. $375, $75, $90. 3. 7 black sheep.
4. 33 girls. 5. 315 passed. 6. .12075.

Exercise 137. Page 189

1. 32, 3.2, .32. 2. 24, .24, .024. 3. 1.2, .12, .012.
4. 1.51, 1.411, .015. 5. 18.5625. 6. 1.1675.
7. 3.642. 8. 19.5025. 9. 3.449.
10. 37.072.

Exercise 138. Page 190

3. 4. 4. .8. 5. 2. 6. 21. 7. 50. 8. .7.
9. 40. 10. 4. 11. 20. 12. 210. 13. 5. 14. 7.
15. 400. 16. 4. 17. 2. 18. 21. 19. 500. 20. 70.
21. $9.10. 22. 35 bu. 23. $446.448. 24. $33.6525.

Exercise 139 — Oral

Exercise 140. Page 193

1. 25%. 2. 40%. 3. 25%. 4. 33⅓%.
5. 72c. 6. $22. 7. 33⅓%. 8. 300.
9. 200 pupils present.

Exercise 141. Page 193

1. $40; 20 mi.; 14.5 lb. 2. 700. 3. 200 mi.
4. 225 head. 5. $9180. 6. 640. 7. 1400.
8. 31¼%. 9. $5.50.

Exercise 142. Page 194

1. $77\frac{1}{2}\%$.
2. $85\frac{5}{7}\%$.
3. $97\frac{9}{13}\%$.
4. (i) .05, .1, .15, .2, .25, .75.
(ii) 50%, 52.5%, 62.5%.
(iii) $\frac{1}{8}, \frac{1}{4}, \frac{3}{16}, \frac{1}{2}, \frac{1}{4}$.
5. $5192.
6. $52.80.
7. $33\frac{1}{3}\%$.
8. $\$1111\frac{1}{9}$.
9. 75

Exercise 143. Page 196

1. $13.60.
2. $32.40.
3. $6.72.
4. $12.75.
5. $6.72.
6. $6.66.
7. $\$4.71\frac{1}{4}$.
8. $\$6.26\frac{2}{3}$ (in interest or $84.564 to pay debt).
9. $73.20 + $105.60 + $195.00 + $150.00 = $523.80.
10. $4736.

Exercise 145. Page 197

1. $9.92.
2. $8.40.
3. $\$2.12\frac{12}{13}$ or $2.13.
4. $5.60.
5. $\$2.33\frac{3}{7}$ or $2.34.
6. $\$21.02\frac{6}{7}$ or $21.02.
7. $11.52.

Exercise 146. Page 198

1. Four thousand eight hundred and fifty-nine.
Four thousand and fifty-nine.
Four thousand five hundred and nine.
Four thousand five hundred and ninety.
Four thousand and nine.
2. Six thousand and seventy-eight.
Seven thousand and sixty-eight.
Eight thousand seven hundred and six.
Seven thousand six hundred and eight.
Six thousand seven hundred and eight.
3. Nine thousand one hundred.
Nine thousand and eleven.
Nine thousand and one.
One thousand nine hundred, or nineteen hundred.
One thousand and ninety, or ten hundred and ninety.

4. Thirty-six thousand seven hundred and eighty-nine.
Thirty-seven thousand six hundred and eighty-nine.
Thirty-eight thousand seven hundred and sixty-nine.
Thirty-nine thousand eight hundred and seventy-six.
Thirty-six thousand eight hundred and ninety-seven.
5. Seventy thousand and eight.
Seventy thousand eight hundred.
Seventy thousand eight hundred and nine.
Seventy-eight thousand and nine.
Seventy thousand eight hundred and ninety.
6. Seventy-five thousand eight hundred and six.
Seventy-five thousand and eighty-six.
Seventy thousand five hundred and eighty-six.
Seventy-eight thousand and fifty-six.
Fifty-eighty thousand seven hundred and sixty.
7. Ninety thousand and five.
Ninety-five thousand.
Ninety-five thousand five hundred and fifty-five.
Fifty-five thousand and ninety.
Fifty thousand nine hundred and five.
8. Seven hundred and thirty thousand six hundred and one.
Nine hundred thousand and five.
Nine hundred and five thousand and fifty.
One hundred and ninety thousand and seventy-six.
Nine hundred and ten thousand.

Exercise 147. Page 198

1. 149 ; 308 ; 974.
2. 200 ; 420 ; 694.
3. 735 ; 960 ; 406.
4. 6006 ; 4300 ; 9080.
5. 40004 ; 44400 ; 90090.
6. 99000 ; 99900 ; 90900.
7. 9090 ; 99009 ; 99099.
8. Oral.

Exercise 148. Page 199

1. 100 ; 999.
2. 9999999 ; 1000000.
3. Seven hundred and eighty-nine million six hundred and forty-three thousand nine hundred and sixty-five.

Nine hundred and eighty-seven million one hundred and eight.

4. 28000 ; 302000 ; 900000.
5. 49999 ; 89908 ; 479890.
6. 5005050 ; 300003300 ; 500000006.
7. XIX ; XXIV ; XLIX ; LXXXIV ; XCIX , XLIV ; XXXIX ; XCIV ; CLXXXVII ; CCVIII ; DCCLXXXI ; CMLXII ; CMXCIX ; CDXLIV ; CDIX ; CMIV.
8. 555 ; 1604 ; 1819 ; 1090 ; 1902.
9. 999 = CMXCIX

Exercise 149. Page 200

1. 33935. 2. 36187. 3. 36708.
4. 28343. 5. 32927. 6. 251776.
7. 252364. 8. 94903. 9. 76931.

Exercise 150. Page 200

Horizontal lines

1. 4938. 2. 3094. 3. 5793. 4. 5714. 5. 5475.
6. 4236. 7. 3892. 8. 5460. 9. 5847. 10. 5371.
11. 6508. 12. 5235. 13. 5789. 14. 4125. 15. 4849.

Vertical lines–

8988 ; 9589 ; 10312 ; 9714 ; 10301 ; 8030 ; 9074 ; 10288.
Total–76326.

Exercise 151. Page 201

1. 7164. 2. $11165. 3. 162 degrees. 4. 699921.
5. A, $375; B, $834; C, $3537; D, $4762; $10000 - $9508 = $492.
6. 323637. 7. 1782 mi.
8. Deposits, $2722.49; withdrawals, $1252.38; balance, $1470.11.
9. 6699 ; 3797 ; 700 ; 2697 ; 2816 ; 2810 ; 91. Sum = 19610.
10. 8280694. 11. 98993.

Exercise 152. Page 202

1. 976100.
2. $56962.75.
3. 457086296631.
4. 2082134.
5. $397143.16.
6. 86081703319.
7. 3215933.
8. $109157.84.
9. 45850097562.
10. 42.
11. 200.
12. $14.
13. $114\frac{1}{4}$.
14. 22.
15. $189\frac{1}{4}$.
16. $22\frac{1}{3}$.
17. $110\frac{2}{7}$.
18. 90.

Exercise 153. Page 202

1. $288.
2. $954.
3. 608 sheep.
4. 87 times.
5. 86, i.e., 2 times 43.
6. $53.
7. $31250.
8. 24.
9. Cost = ($6750 − $990 − $1530) = $4230.
   Cost per bu. = $4230.00 ÷ 9000 = 47c.
10. $7.20.
11. 9319200 + 9317550 + 7687659375 + 11295 = 7706307420.
12. $1116.
13. 365 ac.
14. 3 bu.
15. 1071 bu.
16. 580 ac., $61 per ac.
17. 29 ac. at $95 = $2755.
    75 " $112 = $8400.
    46 " $96 = $4416.

Total selling price, $15571.
Gain = $15571 − $12000 = $3571.

18. 663 mi.

Exercise 154. Page 204

1. 92 pence.
2. 1104 farthings.
3. £29 15*s*. 5*d*.
4. £309 5*s*.
5. 2406 pence.
6. 560 pence.
7. 65296 oz.
8. 86 oz.
9. 13 cwt. 52 lb. 13 oz.
10. 2 tons 7 cwt. 74 lb. 1 oz.
11. 4933 oz.
12. 25 tons 16 cwt. 49 lb.
13. 65142 in.

14. 1 mile 1091 ft. = 1 mile 66 rd. 2 ft.
15. 36 ft.
16. 232 fathoms 4 ft.
17. 12 ac. 77 sq. rd.
18. 117900 sq. in.
19. 4 cu. ft. 1557 cu. in.
20. 60 cords 9 cu. ft.
21. 662400 sec.
22. 120 bu. 2 qt.
23. 678 pints.
24. 2311 pints.
25. 61 bu. 25 lb.
26. 1 wk. 2 da. 2 hr. 14 min. 53 sec.
27. 6739740 sec.
28. 12 cords 40 cu. ft.
29. \$120.57$\frac{1}{8}$.

Exercise 155. Page 204

1. 174 lb. 3 oz.
2. 74 cwt. 21 lb. 3 oz.
3. 88 rd. 5 yd. 1 ft. 6 in.
4. 6 wk. 3 da. 6 hr. 50 min. 33 sec.
5. 22 rd. 2 yd. 8 in.
6. 11 months.
7. 19 mi. 280 rd. 4 yd. 1 ft. 6 in.
8. 58 ac. 146 sq. rd. 25 sq. yd. 2 sq. ft. 36 sq. in.
9. 142 bu. 2 pk. 5 qt.
10. 31 gal. 2 qt. 1 pt.

Exercise 156. Page 205

1. 90 cwt. 80 lb.
2. 50 lb. 2 oz. 7 dwt. 3 gr.
3. 75 da. 23 hr. 34 min. 40 sec.
4. £600 9*s.* 6$\frac{3}{4}$*d.*
5. 249 tons 5 cwt. 25 lb.
6. 13 oz. 12 dwt. = 1 lb. 1 oz. 12 dwt.
7. This question should read (£61 18*s.* 4*d.*) ÷ 4 and the result is £15 9*s.* 7*d.*
8. 12 lb. 9 oz. 15 dwt. 18 gr.
9. 16 tons 2 cwt. 38 lb.
10. $\frac{36}{141}$ pt. = $\frac{12}{47}$ pt.
11. 2 cu. yd. 6 cu. ft. 960 cu. in.
12. 10$\frac{3}{37}$.
13. 11$\frac{489}{576}$.

Exercise 157. Page 206

1. Decimal fractions are $\frac{5}{1000}$; .106; 4.325; $\frac{9}{10}$.
   Common fractions are $\frac{3}{8}$; $\frac{1}{2}$; $\frac{4}{7}$.
2. 4$\frac{47}{80}$.
3. 177$\frac{19}{24}$.
4. 46$\frac{11}{27}$; 267$\frac{5}{96}$; 5$\frac{4683}{10000}$.

5. $1\frac{1}{3}$; $\frac{7}{24}$; $\frac{3}{20}$; $2\frac{1}{2}$; $9\frac{3}{8}$; $\frac{28}{37}$; 25; $\frac{77}{1800}$.
6. $\frac{113}{199}$; $\frac{73}{560}$; $1\frac{66}{479}$; $\frac{11084}{11577}=\frac{652}{681}$.
(H. C. F. of 11084 and 11577 = 17.)
7. 452; 13140; 5631; 5530; 7018.
8. $16\frac{1}{8}$.

Exercise 158. Page 206

1. $\frac{25}{144}$; $\frac{7}{180}$; $2\frac{1}{24}$.
2. .01.
3. .00007208.
4. $\frac{7}{10}$; $\frac{6}{1000}=\frac{3}{500}$; $\frac{336}{1000}=\frac{42}{125}$; $\frac{125}{1000}=\frac{1}{8}$.
5. 557211. (Pupils should be instructed not to reduce the multiplicand to an improper fraction.)
6. \$35.50.
7. 74 lb. 7 oz.
8. $\frac{9}{11}$; $\frac{5}{7}$; $\frac{41}{57}$; $\frac{25}{28}$; $\frac{4}{5}$; $\frac{2}{3}$.
9. (*a*) $\frac{2}{3}$ (*b*) 4.
10. $\frac{71}{120}$; $7\frac{1}{2}$; $40\frac{7}{8}$.
11. \$36.90 (six working days per week).
12. 24 da.
13. 18 bags.
14. $\frac{2}{3}$ of $\frac{9}{16}=\frac{3}{8}$; $\frac{3}{8}, \frac{2}{5}, \frac{3}{7}, \frac{7}{15}, \frac{13}{25}=\frac{1575}{4200}, \frac{1680}{4200}, \frac{1800}{4200}, \frac{1960}{4200}$ and $\frac{2184}{4200}$.
15. (*a*) \$7.74; (*b*) \$9.05; (*c*) \$7.41$\frac{1}{2}$ or \$7.42.
16. 35 tons, 18 cwt. 8 lb.

Exercise 159. Page 208

1. \$211.20; \$190.08.
2. $12\frac{1}{2}\%$.
3. \$247.
4. $22\frac{1}{17}$c.
5. \$44.53$\frac{1}{2}$; \$22.26$\frac{3}{4}$; \$111.33$\frac{3}{4}$; \$11.13$\frac{3}{8}$.
6. \$250.00 + \$12.49$\frac{23}{73}$ or \$262.49.
7. \$600.00 + \$7.29$\frac{63}{73}$ or \$607.30.
8. \$15.71$\frac{17}{73}$.

PART II

Exercise 1. Page 9

1. 361280202. Three hundred and sixty-one million, two hundred and eighty thousand, two hundred and two; 275248604. Two hundred and seventy-five million, two hundred and forty-eight thousand, six hundred and four; 720006. Seven hundred and twenty thousand and six. 201204000. Two hundred and one million, two hundred and four thousand;

   2002012. Two million, two thousand and twelve.

2. 607070. Six hundred and seven thousand and seventy. 8100180. Eight million, one hundred thousand, one hundred and eighty;

   10150105. Ten million, one hundred and fifty thousand, one hundred and five;

   1015010. One million, fifteen thousand and ten; 101501. One hundred and one thousand, five hundred and one; 7007022. Seven million, seven thousand and twenty-two.

3. 70070220 Seventy million, seventy thousand, two hundred and twenty;

   700702202. Seven hundred million, seven hundred and two thousand, two hundred and two;

   770077077. Seven hundred and seventy million, seventy-seven thousand, and seventy-seven;

   500050005. Five hundred million, fifty thousand and five; 6076076. Six million, seventy-six thousand, and seventy-six.

4. Seven hundred and seven, and seven tenths; Eight hundred and fifty, and seventy-nine hundredths; Five thousand six hundred and ninety-five, and six hundredths;

   Four hundred and seventy-three, and six hundred and twenty-eight hundredths.

5. Five hundred and sixty-four, and eighteen hundredths;
Seven thousand, eight hundred and forty, and six hundredths;
Four thousand and five, and seven hundredths;
Thirty-six thousand, and forty-one hundredths.

6. Three thousand, and seventy-one hundredths;
Nine hundred and one thousand and seven.
Seven hundred and twenty thousand and nine;
One thousand, eight hundred and twenty, and one hundred and six thousandths.

7. Thirty-one thousand four hundred, and six hundredths;
Fifty thousand, and six hundred and four thousandths;
Thirty-six thousand, and one hundred and seven thousandths.

8. Four hundred and four thousand and four, and one thousandth;
Four hundred and forty thousand, and four hundredths;
Forty thousand and four, and four thousandths.

9. Five hundred million, five hundred thousand, five hundred, and one hundredth;
Five hundred and five million, five thousand and fifty, and five hundredths;
Five million, five thousand, and five hundred and five thousandths.

10. Seventy-seven million, seven thousand, seven hundred, and seven hundred and seven thousandths;
Eight billion, ninety-seven million, eighty-eight thousand and seven, and six hundredths;
Nine hundred and four million, four hundred and forty, and one thousandth.

11. Five hundred and sixteen million, three hundred and sixty-two thousand, and seventy-six, and four hundred and one thousandths;
Seven hundred million and one, and two thousandths;
Nine billion, two hundred million, seven hundred thousand, seven hundred, and seven hundred and four thousandths.

12. Hundreds, thousands, millions, billions, trillions.

13. (*a*) 800008; (*b*) 800080; (*c*) 800800. (*d*) 8808; (*e*) 8880; (*f*) 80800.

14. (*a*) 1001101; (*b*) 1010110; (*c*) 1101001; (*d*) 1100001; (*e*) 1000001.

15. $\frac{1}{10}=.1$; $\frac{1}{100}=.01$; $\frac{1}{1000}=.001$; $\frac{6}{10}=.6$; $\frac{66}{100}=.66$; $\frac{66}{1000}=.066$; $\frac{7}{10}=.7$; $\frac{70}{100}=.70$; $\frac{5}{1000}=.005$; $\frac{405}{1000}=.405$; 2000.0006.

16. 5.5; 4.04; 6.006; 10.11; 11.074; 19.0095.

Exercise 2. Page 10

1. CDXLIV, CMXCIX, CMXLIX.
2. MCDXCIX, MMCDIX, MCMII.
3. MDCCCLXXV, MCMXIV, MDCCCXXXIX.
4. MMMDCCCXLIX, MDCCCXLIV, MCM.
5. MDCCXCIV, MCDXCVII, MCMXC.
6. MMCCCXLV, MCMXCIX, MCDXLIV.
7. 99, 1129, 990, 994, 554.
8. 1009, 1509, 1400, 1339, 544.
9. 1889, 1540, 1490, 1995
10. 10, 20, 30, 90, 100, 110, 120, 130, 190, 200, 210, 220, 230, 290, 300, 310, 320, 330.
11. (1902 × 1814) ÷ 317 = 10884.

Exercise 3. Page 11

1. 11277349. (2) 11871791. (3) 10892561. 4. 11527333.

5. First week, $420.52; second week, $417.44; third week, $453.62; fourth week, $368.00; fifth week, $395.36; sixth week, $440.14; Monday, $449.96; Tuesday, $355.46; Wednesday, $408.49; Thursday, $458.24; Friday, $426.98; Saturday, $395.95. Total, $2495.08.

6. 172907090947.
7. 365338049326.
8. 743712815593.
9. 150550736973.
10. 102442; the remainders are 809509; 730946; 652383; 573820; 495257; 416694; 338131; 259568; 181005; 102442.
11. 365491989306236.
12. 77924824040448.
13. 1739946670192879.
14. 653452078096964.

Exercise 4. Page 12

1. Quotient = 128640 and remainder = 2329.
2. " = 5479 " " = 10728.
3. " = 2839 " " = 95122.
4. " = 240870 " " = 1240.
5. No. to be added is 5. $\left(\frac{399 \times 95}{133} - 280 = 5\right)$.
6. $\frac{70541 - 54678}{29} = 547.$ 7. $827658432 \div (12 \times 144) = 478969.$
8. Rem. = 8966. Dividend = $8967 \times 8967 + 8966 = 80416055.$
9. Divisor = $(87911123 - 2743) \div 12434 = 7070.$

Exercise 5. Page 13

1. Gain is $50.92. 2. Cheaper to hire boys by $312.
3. 14 gal. of water added.
4. No. tons bought = $\{(965 - 160 \times 5) \div 3\} + 160 = 215.$
5. Gain = $311.15.
6. After 20 da. the provisions would last the 720 men 30 da. longer, 2160 men for 10 da.

   ∴ No. *additional* men = 2160 − 720 = 1440.
7. Value of sheep = $\$\frac{455 \times 3 \times 5}{7 \times 2 \times 26} = \$18.75.$
8. Had he worked 40 da. he would have received $\$1.50 \times 40 = \$60.$ Every day he was idle he lost his wages and 50c. for board, or $2. ∴ he was idle $\frac{20}{2}$ da. = 10 da.; and ∴ he worked 30 da.

Exercise 6. Page 14

1. $236\frac{1}{2}$ yd.; $557\frac{1}{2}$ yd.
2. $95\frac{1}{11}$ rd. = 95 rd. 1 ft. 6 in.; $151\frac{31}{33}$ rd. = 151 rd. 5 yd. 6 in.; $5\frac{141}{198}$ rd. = 5 rd. 3 yd. 2 ft. 9 in.
3. 444617 in. 4. $11041\frac{1}{4}$ sq. yd.; $2287\frac{3}{4}$ sq. yd.
5. $29\frac{51}{121}$ sq. yd. = 29 sq. rd. 12 sq. yd. 6 sq. ft. 108 sq. in.; $165\frac{35}{121}$ sq. rd. = 165 sq. rd. 8 sq. yd. 6 sq. ft. 108 sq. in.
6. 32897152 sq. in. 7. 5 mi. 311 rd. 5 yd. 2 in.
8. 92 mi. 126 rd. 2 yd. 9. 16 mi. 162 rd. 3 yd. 5 in.
10. $3\frac{1}{5}$ mi. per hr. 11. 198.

Exercise 7. Page 15

1. No. da. from end of June 15 to beginning of Dec. 31 = 198.
" hr. " " " " = 4752.
But there are 12 hr. on June 15, and 9 hr. on Dec. 31 to be added; ∴ No. hr. required = 4773.

3. No. mi. per hr. = $\frac{80 \times 60 \times 60}{4 \times 1760} = 40\frac{10}{11}$.

3. Cost of hay = $\$\frac{10 \times 15 \times 7 \times 40 \times 13}{2000} = \$273$.
" " oats = $\$\frac{40 \times 12 \times 7 \times 40 \times 13}{32} = \$546$.
Total cost = \$819.

4. 12 ft. 6 in.

5. $22\frac{2}{3}$ ft. or 22 ft. 8 in. board measure; 10648 cu. in.

6. A's farm 25 ac. 19 sq. rd. 7 sq. yd.
B's " = 6 " 44 " 24 " 3 sq. ft. 135 sq. in.
C's " = 47 " 82 " 19 " 5 " 135 "
Total = 78 " 146 " 20 " 7 " 90 "

7. No. mi. = $\frac{3 \times 160 \times 30\frac{1}{4} \times 9 \times 144}{9 \times 12 \times 3 \times 1760} = 33$.

8. No. revolutions = $\frac{30 \times 85 \times 60}{46} = 3326\frac{2}{23}$.

9. No. bu. = $\frac{1785}{84}$. No. bu. to ac. = $2\frac{1}{2}$; ∴ No. ac. $\frac{1785 \times 2}{84 \times 5} = 8\frac{1}{2}$.

Exercise 8. Page 16

1. No. bu. = $(8 \times 5 \times 3 \times 48) \div 60 = 96$. 2. \$13.40. 3. \$145.94.

4. 6 mi. per hr. 5. No. bu. oats = $\frac{(68 \times 48) + (51 \times 60)}{34} = 186$.

6. Cost of land = $\$(300 \times 40 \times 25) \div 160 = \$1875$.
" " fence = \$195. Total cost = \$2070.

7. \$14.56. 8. \$37.50.

Exercise 9. Page 17

1. $1\frac{1}{4}$ mi. 2. No. lb. butter = $\frac{1\frac{1}{2} \times 12\frac{1}{2} \times 7 \times 16}{25} = 84$.

3. 12 lb. 4. \$2.80. 5. 3627 cords.

6. No. cu. yd. = $\frac{200 \times 20 \times 33}{27} = 4888\frac{8}{9}$.

7. \$120. 8. (*a*) \$4.80; (*b*) \$2.50.

9. Value of barley $=$ \$$(.40 \times 4032) \div 48 =$ \$33.60. Value reckoned as rye $=$ \$$(.49 \times 4032) \div 56 =$ \$35.28. Farmer's gain $=$ \$1.68.

10. No. bu. in bin $= 12 \times 12 \times 12 \times \frac{100}{128}$.

$$\text{Weight of 1 bu.} = \frac{45900 \times 128}{12 \times 12 \times 12 \times 100} \text{ lb.} = 34 \text{ lb.}$$

Exercise 10. Page 18

1. 990 ft. = 330 yd. = 60 rd.; 10 chains.
2. \$422.40. 3. $733\frac{1}{3}$ c. yd. 4. 128 ft.
5. $\frac{1375}{3456}$ lb., or $6\frac{79}{216}$ oz.
6. $\frac{1}{7}$ of 45 ac. = 6 ac. 68 rd. 17 sq. yd. 2 sq. ft. $82\frac{2}{7}$ sq. in.
7. 1000000 in. = 15 mi. 250 rd. 2 yd. 2 ft. 4 in.
Difference = 84 " 69 " 2 " 2 " 2 "

Exercise 11. Page 18

1. No. ac. = 24. Length = 120 rd.
2. 27900 shingles. 3. \$51.28.
4. Perimeter = 1100 yd. = 3300 ft. Cost = \$247.50.
5. 3520 cu. yd. 6. 10 mi. 7. \$71.28. 8. 15 times.
9. 8640 cu. ft.; 216 cu. ft. per pupil.

Exercise 12. Page 19

1. $\frac{9}{10}$ ac. 2. $\frac{288}{605}$ ac. 3. $14\frac{2}{5}$ ac.
4. 320 ac. Area enclosed is half the section
5. 50 ac.; 360 rd.; $137\frac{1}{2}$ hr. or $13\frac{3}{4}$ da. of 10 hr. each.
6. (*a*) Breadth = 24 rd.; perimeter = 128 rd.
(*b*) Area = 16 ac.; " = 56 ch.
(*c*) " = $3\frac{3}{4}$ " length = 30 rd.
(*d*) Length = 20 ch.; perimeter = 50 ch.
7. (*a*) Cubic content = 72 cu. ft.; surface = 108 sq. ft.
(*b*) Length = 5 ft.; " = 184 "
(*c*) Breadth = $2\frac{4}{11}$ ft.; " = $88\frac{10}{11}$ "
8. \$2156. 9. Depth = 20 ft.
10. (*a*) Area = 484 sq. yd.; (*b*) area = 16 sq. rd.

Exercise 13. Page 21

1. $10=2\times5$; $35=5\times7$; $14=2\times7$; $77=7\times11$; $33=3\times11$; $55=5\times11$.
2. $45=3\times3\times5$; $30=2\times3\times5$; $42=2\times3\times7$; $70=2\times5\times7$; $66=2\times3\times11$.
3. $144=2\times2\times2\times2\times3\times3$; $90=2\times3\times3\times5$; $75=3\times5\times5$; $72=2\times2\times2\times3\times3$; $84=2\times2\times3\times7$.
4. 61, 67, 71, 73.
5. Common factors of 16 and 24 are 2, 4, 8; of 24 and 30 are 2, 3, 4, 6; of 36 and 42 are 2, 3, 6; of 70 and 60 are 2, 5, 10; of 25 and 35 the common factor is 5.
6. 2 and 5; 3 and 7; 10 and 13; etc. *N.B. Unity is a factor of all numbers.*
7. H.C.F. of 27 and 36 is 9; of 72 and 64 is 8; of 45 and 75 is 15; of 100 and 75 is 25; of 125 and 75 is 25; of 108 and 81 is 27.
8. The number is divisible by 3 when the sum of its digits is divisible by 3.
9. 111, 243, 510, 321, *or* 114, 246, 513, 324, etc.
10. $360=2\times2\times2\times3\times3\times5=2^3\times3^2\times5$; $560=2^4\times5\times7$; $845=5\times13^2$; $1640=2^3\times5\times41$.
11. No. $=2\times3\times5\times7\times11=2310$.
12. $249984=2^7\times3^2\times7\times31=(2\times31)\times(3^2\times7)\times2^6=62\times63\times64$.

Exercise 14. Page 22

1. 3, 4, 5.
2. 45.
3. 12.
4. 24.
5. 8.
6. $3.60 per bbl.
7. 21c. per doz.
8. $10.
9. $1.20.
10. 4.
11. $28\frac{1}{2}$c.
12. 24.
13. 20160, i.e., $\dfrac{8\times15\times24\times42\times65\times77}{2\times3\times5\times7\times11\times13}$.

Exercise 15. Page 23

1. $248=2^3\times31$; $356=2^2\times89$; common factors are 2 and 4, each of which is a factor of 108.

2. Write the numbers 27 and 42 having a common factor 3 which is also a factor of their sum (69), and of their difference (15).
3. Take the numbers 6 and 20 having a common factor 2. Take any multiple of 6, say 4 times 6 or 24; the factor 2 is a factor of the difference between 24 and 20.

Exercise 16. Page 24

1. 23. 2. 37. 3. 41. 4. 56. 5. 45. 6. 61.
7. 42. 8. 11. 9. 813. 10. 630. 11. 928. 12. 1249.

Exercise 17. Page 24

1. 7. 2. 21. 3. 7. 4. 607.

Exercise 18. Page 25

1. 40 ft. 2. 6 ac.
3. Lots are 60 ft. wide; A has 10, B has 12, and C has 15.
4. A makes 1 circuit in 30 da., B in 20 da., C in 24 da., and D in 15 da. They will all be together at the starting point in 120 da. (L.C.M. of 30, 20, 24 and 15) when A will have made 4 rounds; B, 6; C, 5; D, 8. 5. 12 in.
6. Cost of each horse is \$89; A bought 7 and B 12.
7. H. C. F. of (64610 − 27) and (72204 − 23) is 3799. The smallest divisor must be greater than 27 and a factor of 3799, and is 29. (3799 = 29 × 131). 8. 37. 9. 7 ft.

Exercise 19. Page 26

1. Multiples of 5 are 10, 15, 20, etc.; of 7 are 14, 21, 28, etc.; of 8 are 16, 24, 32, etc.; of 9 are 18, 27, 36, etc.; of $2\frac{1}{2}$ are 5, $7\frac{1}{2}$, 10, etc.; of $3\frac{1}{4}$ are $6\frac{1}{2}$, $9\frac{3}{4}$, 13, etc.
2. 21 is a multiple of 3 and 7; 35 of 5 and 7; 55 of 5 and 11; 63 of 7 and 9; 77 of 7 and 11.
3. 14, 28, 42, 56. 4. 28, 66, 40, 18, 24, 30, 36.
5. 12, 24, 36, 48, 60; 12, 24, 36, 48, 60; 60, 120, 180, 240, 300; 60, 120, 180, 240, 300. 6. 180. 7. 2400. 8. 600.
9. 1440. 10. 2520. 11. 1680. 12. 21, 41, 61. 13. 32, 62, 92.

Exercise 20. Page 26

1. L.C.M is 149688; G.C.M. is 119. 2. (*a*) 354025; (*b*) $300.
3. $435. (L.C.M. of sums = $420.)
4. Width of room = 45 ft. (L.C.M. of $\frac{3}{4}$ yd., 1 yd., $1\frac{1}{4}$ yd., $1\frac{1}{2}$ yd.). No. of strips of carpet, $\frac{3}{4}$ yd. wide = 20. No. yd. carpet = $\frac{60}{3} \times 20 = 400$. Cost = $440.
5. 28 bu. wheat. (L.C.M. of 60, 56, 48 = 1680; $\frac{1680}{60} = 28$).
6. They strike in unison every 84 sec., ∴ first time they strike in unison is 1′ 24″ after 12. In 7 min. they are in unison 5 times.
7. Left foot is down every 60″, 66″, 72″. Distance when left feet are first down together = L.C.M. of 60″, 66″, 72″ = 3960″. ∴ No. times in 1 mile $= \frac{5280 \times 12}{3960} = 16$.
8. 6. (L.C.M. of 48 and 56 = 336; 336 ÷ 56 = 6).
9. Same cogs together in every 8 revolutions of the larger wheel, or 8 times per sec. Required No. times = 6 × 8 × 60 × 60 × 6 = 1382400.
10. Other No. = (120120 × 210) ÷ 2730 = 9240. (Show that the product of the G.C.M. and the L.C.M. of two Nos. = the product of the Nos.).

Exercise 21. Page 28

1. $360. 2. 123. 3. 12. 4. 6.
5. 25. 6. 42. 7. 1584.
8. 23 ft. long, 15 ft. wide. 9. $10. 10. $1.70.
11. 432. 12. A gets $4; B, $6; C, $12. 13. 81.
14. 16 in. 15. $45.

Exercise 22. Page 29

1. A, $8; B, $12. 2. 5, 7, 11, 13.
3. Length = 18 ft.; breadth = 9 ft.
4. G.C.D. of 560 and 840 = 280. All the factors of 280 will be common factors of 560 and 840.

5. 9. 6. 33. 7. 309. 8. 165 cu. ft. 9. $132.84.

10. 825371. Divisor = 908.

11. Rails are 13 ft. long. No. = 2976.

12. 1650 (L.C.M. of 25, 30, 55).

13. $8400 = 2^4 \times 3 \times 5^2 \times 7$; $3820 = 2^2 \times 5 \times 191$; $1380 = 2^2 \times 3 \times 5 \times 23$.
$\therefore$ G.C.M. $= 2^2 \times 5$, and L.C.M. $= 2^4 \times 3 \times 5^2 \times 7 \times 23 \times 191$.

14. $7\frac{1}{2}$ lb. 15. 1938. Each tile is 10 in. square.

16. Divisors of 360 are 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180.

17. $3252 = 2^2 \times 3 \times 271$; $4248 = 2^3 \times 3^2 \times 59$; (*a*) 2, 3, 4, 6, 12; (*b*) $2^2 \times 3 = 12$; (*c*) $2^3 \times 3^2 \times 59 \times 271 = 1151208$.

18. 336 cu. ft.; 10080 lb. 19. $6\frac{2}{3}$ ft. 20. $3.12.

21. 28 mi. L.C.M. of $9\frac{1}{3}$ ft. and $11\frac{2}{3}$ ft. $= \frac{140}{3}$ ft.
Dist. $= \frac{140}{3} \times \frac{3168}{5280}$ mi. $= 28$ mi.

22. No. cu. ft. lumber $= (6 \times 4\frac{1}{2} \times 3\frac{1}{3} - 5\frac{5}{6} \times 4\frac{1}{3} \times 3\frac{1}{6})\quad 90 - 80\frac{5}{108}$
$= 9\frac{103}{108}$.
No. ft. lumber, board measure $= 9\frac{103}{108} \times 12 = \frac{1075}{9}$.
Cost per M. $= \$3.10 \times \frac{9}{1075} \times 1000 = \$25.95\frac{15}{43}$.

Exercise 23. Page 31

11. $\frac{133}{12}$. 12. $\frac{99}{8}$. 13. $\frac{431}{12}$. 14. $\frac{1402}{7}$.
15. $\frac{1199}{23}$. 16. $\frac{1839}{19}$. 17. $\frac{1099}{11}$. 18. $\frac{7741}{99}$.

Exercise 24. Page 31

1. 9 boys. 2. $\frac{22}{9}$. 3. 23 mi.
4. $\frac{112}{7}$; $\frac{253}{11}$. 5. 47. 6. 2.
7. $\frac{128}{32}$; $\frac{256}{32}$; $\frac{512}{32}$. 8. $\frac{11}{14}$; $\frac{366}{46}$. 9. 60.
10. 15. 11. 43. 12. 5.

Exercise 25. Page 32

7. $6\frac{2}{7}$. 8. $15\frac{3}{8}$. 9. $8\frac{1}{11}$. 10. $24\frac{1}{4}$.
11. $12\frac{4}{13}$. 12. $16\frac{13}{15}$. 13. $65\frac{7}{15}$. 14. $16\frac{1}{5}$.
15. 13. 16. 28. 17. [illegible]$1\frac{13}{15}$. 18. $32\frac{54}{185}$.

Exercise 26. Page 33

1. $\$\frac{7}{8} = 87\frac{1}{2}$c.
2. \$1.75.
3. 9 pies.
4. 13 eighths ($\frac{13}{8}$).
5. $16\frac{1}{3}$ gal.
6. 65 mi.
7. $55\frac{3}{4}$ bu.
8. Length = $18\frac{3}{4}$ ft.; width = 13 ft.
9. $\frac{5000}{7}$ rd. = 2 mi. $74\frac{2}{7}$ rd.
10. 440, *or* 441 if one at each end.
11. No exact number of lb.-weights will balance; 46 lb.-weights, one 8-oz. weight and one 4-oz. weight would do, i.e., $46\frac{3}{4}$ lb.
12. $33\frac{1}{3}$ hr.; 4 additional boxes.

Exercises 27, 28 - Oral

Exercise 29. Page 35

5. $\frac{7}{8}$.
6. In lowest terms.
7. $\frac{1}{5}$.
8. $\frac{2}{5}$.
9. $\frac{3}{8}$.
10. $\frac{5}{7}$.
11. $\frac{4}{7}$.
12. $\frac{1}{10}$.
13. $\frac{1}{4}$.
14. $\frac{11}{12}$.
15. $\frac{13}{27}$.
16. $\frac{7}{9}$.
17. $\frac{21}{71}$.
18. $\frac{3}{4}$.
19. $\frac{81}{224}$.
20. $\frac{219}{417}$.
21. $\frac{19}{67}$.
22. $\frac{1}{4}$.
23. $\frac{7}{11}$.
24. $\frac{4}{15}$.

Exercises 30, 31, 32 — Oral

Exercise 33. Page 37

1. $\frac{20}{30}, \frac{25}{30}, \frac{18}{30}$.
2. $\frac{4}{8}, \frac{6}{8}, \frac{7}{8}$.
3. $\frac{15}{40}, \frac{36}{40}, \frac{14}{40}$.
4. $\frac{20}{36}, \frac{21}{36}, \frac{22}{36}$.
5. $\frac{30}{36}, \frac{10}{36}, \frac{21}{36}$.
6. $\frac{45}{60}, \frac{42}{60}, \frac{50}{60}$.
7. $\frac{15}{60}, \frac{35}{60}, \frac{33}{60}$.
8. $\frac{25}{30}, \frac{24}{30}, \frac{20}{30}$.
9. $\frac{35}{40}, \frac{30}{40}, \frac{32}{40}$.
10. $\frac{70}{105}, \frac{63}{105}, \frac{75}{105}$.
11. $\frac{75}{105}, \frac{70}{105}, \frac{84}{105}$.
12. $\frac{36}{60}, \frac{40}{60}, \frac{45}{60}$.
13. $\frac{20}{15}, \frac{3}{15}, \frac{8}{15}$.
14. $\frac{15}{6}, \frac{20}{6}, \frac{5}{6}$.
15. $\frac{7}{9}, \frac{39}{9}, \frac{45}{9}$.
16. $\frac{15}{16}, \frac{72}{16}, \frac{1}{16}$.
17. $\frac{147}{168}, \frac{60}{168}, \frac{70}{168}, \frac{52}{168}$.
18. $\frac{44}{77}, \frac{35}{77}, \frac{9}{77}, \frac{462}{77}$.

Exercise 34. Page 37

1. $\frac{15}{35}, \frac{20}{35}, \frac{14}{35}$. In order of magnitude: $\frac{7}{15}, \frac{5}{17}, \frac{5}{8}$.
2. $\frac{162}{216}, \frac{144}{216}, \frac{135}{216}, \frac{126}{216}$. In order of magnitude: $\frac{3}{4}, \frac{6}{9}, \frac{5}{8}, \frac{7}{12}$.
3. $\frac{16}{20}, \frac{15}{20}$; $\frac{4}{5}$ of a field is the greater.
4. $\frac{4}{7}$.
5. $\frac{25}{27}$.
6. $\frac{10}{21}$.
7. $\frac{11}{13}$.
8. $\frac{17}{20}$.
9. $\frac{17}{21}$.
10. $\frac{5}{54}$.
11. $\frac{7}{40}$.

12. Greatest = $\frac{18}{25}$, least = $\frac{12}{35}$. 13. Greatest = $\frac{11}{12}$, least = $\frac{8}{9}$.
14. " = $\frac{4}{5}$, " = $\frac{3}{16}$. 15. " = $\frac{1}{10}$, " = $\frac{3}{4}$.
16. " = $\frac{13}{18}$, " = $\frac{11}{17}$. 17. " = $\frac{3}{5}$, " = $\frac{4}{9}$.
18. $\frac{4}{35}$, $\frac{3}{14}$, $\frac{8}{27}$, $\frac{17}{41}$, $\frac{3}{5}$. 19. $\frac{1}{2}$, $\frac{13}{28}$, $\frac{3}{5}$, $\frac{3}{4}$, $\frac{4}{5}$.
20. $\frac{1}{4} = \frac{15}{60}$, $\frac{2}{5} = \frac{24}{60}$, ∴ reqd. fr. = $\frac{19}{60}$.
21. $\frac{97}{112}$.

Exercise 35 — Oral

Exercise 36. Page 40

1. $\frac{20}{77}$. 2. $\frac{3}{11}$. 3. $\frac{31}{10} = 3\frac{1}{10}$. 4. $\frac{55}{38} = 1\frac{17}{38}$. 5. $\frac{4}{21}$.
6. $\frac{5}{88}$. 7. $\frac{10}{21}$. 8. $\frac{5}{9}$. 9. $\frac{38}{175}$. 10. $\frac{12}{13}$.
11. $\frac{1}{6}$. 12. $\frac{3}{32}$. 13. $\frac{31}{55}$. 14. 3. 15. $\frac{73}{28} = 2\frac{17}{28}$.
16. $\frac{17}{72}$. 17. $\frac{9}{20}$. 18. $\frac{7}{5} = 1\frac{2}{5}$.

Exercise 37. Page 40

1. $\frac{1}{2}$. 2. $\frac{15}{32}$. 3. $ $\frac{9}{25}$ = 36c. 4. $\frac{2}{15}$.
6. $\frac{1}{4}$. 7. $12\frac{1}{2}$ ac. 8. $8750. 9. 24 mi.
10. $6900. 11. $746\frac{2}{3}$. 12. $7035.

Exercise 38. Page 41

1. Four hundred and four millions, forty thousand, four hundred and four, and four hundredths.
2. CDXCIV, CC[illegible] [illegible]IX, CDIV, CMXCIX, MDCCCXCVII, [illegible], MCMXXXV.
3. 52095. [illegible] 2[illegible] m. = 503.)
5. $2700 = 2^2 \times 3^3 \times 5^2$; 10[illegible], 135, 150, 180, 225, 300.
6. No. = $17 \times 19 \times 23 \times 29 \times 31 \times 37$.
7. $360 = 2^3 \times 3^2 \times 5$; $540 = 2^2 \times 3^3 \times 5$; $588 = 2^2 \times 3 \times 7^2$.
   (1) Common factors are 2, 4, 6, 12; (2) G. C. F. = 12.
8. 192 ac. 9. 3. 10. 6 yr. 11. 22.
12. A, $20; B, $35. 13. $1\frac{7}{35}$ bu. 14. $3738.
15. Amount of bill = $49.29; payment = $14.31; balance = $34.98.
16. $\frac{1}{2}\frac{5}{5}$. $\frac{1}{2}$ (sum + difference) = larger No. (denominator);
    $\frac{1}{2}$ (sum − difference) = smaller No. (numerator)
    Give reason for this.

Exercise 39 – Oral

Exercise 40. Page 43

1. $1\frac{5}{12}$. 2. $1\frac{27}{40}$. 3. $\frac{29}{88}$. 4. $1\frac{7}{12}$. 5. $1\frac{39}{40}$. 6. $\frac{107}{200}$.
7. $2\frac{2}{7}$. 8. $2\frac{7}{18}$. 9. $2\frac{9}{35}$. 10. $1\frac{91}{180}$. 11. $3\frac{31}{420}$. 12. $\frac{37}{51}$.

Exercise 41. Page 43

1. $\frac{11}{20}$. 2. $\$\frac{251}{315}$. 3. $\frac{47}{63}$. 4. $2\frac{53}{144}$ t.
5. $\frac{8}{9}$. 6. $2\frac{1}{18}$. 7. $2\frac{107}{240}$. 8. $1\frac{43}{120}$ bbl.
9. $4\frac{1}{4}$ doz. 10. $13\frac{7}{12}$ 11. $9\frac{11}{12}$ yd. 12. $3\frac{3}{4}$ ac.

Exercise 42. Page 44

1. $3\frac{3}{4}$ bu. 3. $10\frac{11}{30}$. 4. $10\frac{7}{18}$. 5. $10\frac{29}{42}$.
6. $5\frac{29}{60}$. 7. $4\frac{109}{288}$. 8. $46\frac{8}{15}$. 9. $22\frac{89}{180}$.
10. $29\frac{7}{18}$. 11. $12\frac{5}{8}$. 12. $21\frac{17}{20}$. 13. $12\frac{5}{8}$.
14. $16\frac{2}{5}$. 15. $14\frac{43}{360}$ gal. 16. $51\frac{33}{40}$ lb. 17. $108\frac{4}{15}$ lb.
18. $53\frac{17}{120}$. 19. $9\frac{49}{120}$ yd. 20. $72\frac{7}{12}$ mi. 21. 861 bu.

Exercise 43—Oral

Exercise 44. Page 47

1. $\frac{4}{35}$. 2. $\frac{17}{45}$. 3. $\frac{13}{36}$. 4. $\frac{1}{20}$.
5. $\frac{1}{16}$. 6. $\frac{2}{105}$. 7. $\frac{7}{60}$. 8. $\frac{409}{700}$.
9. $\frac{77}{171}$. 10. $\frac{7}{18}$. 11. $\frac{1}{15}$. 12. $\frac{1}{42}$.

Exercise 45. Page 47

1. $\frac{1}{3}$. 2. $\frac{1}{3}$. 3. $\frac{31}{45}$. 4. $\frac{13}{30}$.
5. $\frac{19}{72}$. 6. $\frac{3}{4}$; $\frac{1}{60}$. 7. $\frac{47}{100}$. 8. $\frac{11}{35}$.
9. $4.20. 10. $7.25. 11. $9\frac{1}{3}$. 12. $11\frac{7}{12}$ yd.

Exercise 46. Page 48

1. 522. 2. $6\frac{1}{2}$; $3\frac{1}{4}$. 3. 36. 4. $4\frac{2}{5}$.
5. $2\frac{3}{4}$. 6. $1\frac{22}{35}$. 7. $1\frac{1}{90}$. 8. $3\frac{7}{12}$.
9. $2\frac{22}{63}$. 10. $1\frac{7}{12}$. 11. $1\frac{1}{4}$. 12. $2\frac{11}{42}$.
13. $2\frac{53}{56}$. 14. $4\frac{4}{7}$. 15. $14\frac{7}{18}$. 16. $24\frac{11}{18}$.
17. $10\frac{5}{6}$.

Exercise 47. Page 48

1. $2\frac{19}{63}$. 2. $4\frac{1}{2}$. 3. $\frac{7}{18}$. 4. $1\frac{47}{84}$.
5. $3\frac{5}{18}$. 6. $31\frac{7}{9}$. 7. $13\frac{7}{18}$. 8. $\frac{3}{20}$.
9. $\frac{13}{14}$. 10. $7\frac{7}{24}$. 11. $5\frac{26}{35}$. 12. $12\frac{239}{700}$.

Exercise 48. Page 48

1. $18\frac{15}{28}$. 2. $16\frac{9}{16}$ gal. 3. $\frac{49}{60}$. 4. $20\frac{11}{16}$ yd.
5. \$$3\frac{1}{8}$. 6. $101\frac{1}{15}$ ac. 7. $14\frac{13}{30}$ reams. 8. $34\frac{79}{84}$ lb.
9. $36\frac{1}{36}$ mi. 10. \$$38\frac{1}{15}$. 11. \$$98\frac{29}{35}$.
12. In ascending order of magnitude :
$\frac{5}{21}$, $\frac{7}{20}$, $\frac{3}{8}$, $\frac{4}{9}$; $\frac{5}{21}+\frac{4}{9}=\frac{43}{63}$; $\frac{7}{20}+\frac{3}{8}=\frac{29}{40}$; $\frac{29}{40}-\frac{43}{63}=\frac{107}{2520}$.

Exercise 49. Page 49

1. \$$383\frac{7}{12}$. 2. \$1.70. 3. $49\frac{7}{18}$. 4. \$$211\frac{1}{12}$.
5. $107\frac{31}{48}$ gal. 6. $8\frac{31}{36}$. 7. $145\frac{17}{54}$ yd.; \$$403\frac{13}{16}$.
8. $44\frac{9}{16}$ lb. 9. \$3. 10. $10\frac{71}{84}$ gal.
11. \$$177\frac{7}{36}$. 12. $774\frac{1}{63}$ ac.

Exercise 50. Page 50

1. \$12 ; 40 lb. 2. 4 ; 22.
3. *Either* by multiplying the numerator *or* dividing the denominator of the fraction by the whole number.
4. $7\frac{1}{2}$. 5. $6\frac{2}{3}$. 6. $2\frac{1}{4}$. 7. $9\frac{1}{3}$.
8. $37\frac{1}{3}$. 9. $71\frac{1}{2}$. 10. $.26\frac{1}{4}$. 11. $10\frac{2}{19}$.
12. $4\frac{23}{28}$.

Exercise 51. Page 51

1. \$5 ; 5 mi. 2. $\frac{5}{17}$; $\frac{5}{27}$. 3. $\frac{4}{7}$; $\frac{2}{7}$. 4. $\frac{3}{7}$; $\frac{3}{8}$.
5. *Either* by dividing the numerator *or* multiplying the denominator of the fraction by the whole number.
6. $\frac{3}{17}$. 7. $\frac{1}{6}$. 8. $\frac{7}{64}$. 9. $\frac{15}{136}$.
10. $\frac{6}{5}=1\frac{1}{5}$. 11. $\frac{2}{3}$. 12. $\frac{21}{70}$. 13. $8\frac{1}{10}$.
14. $\frac{767}{32}=23\frac{31}{32}$.

Exercise 52 — Oral

Exercise 53. Page 52

1. 15. 2. 40. 3. $37\frac{1}{2}$. 4. $54\frac{1}{4}$.
5. $\frac{18}{25}$. 6. $\frac{3}{50}$. 7. $\frac{3}{8}$. 8. $\frac{7}{44}$.
9. 1. 10. $\frac{19}{65}$. 11. $\frac{7}{18}$. 12. $\frac{29}{65}$.
13. $\$\frac{91}{256}$. 14. $\$\frac{5}{8}$.
15. A, $281.25; B, $225; C, $303.75; total = $810.

Exercise 55. Page 52

1. $17\frac{5}{28}$. 2. $49\frac{1}{3}$. 3. 290. 4. 1320.
5. $7390\frac{3}{4}$. 6. $20\frac{1}{2}$.

Exercise 56. Page 52

1. $14\frac{13}{840}$. 2. $93. 3. $\$1.66\frac{2}{3}$. 4. $\frac{22}{25}$.
5. $394\frac{38}{45}$. 6. $\frac{38}{1125}$. 7. $\$12.10\frac{1}{4}$. 8. $95.60.
9. $\$1.95\frac{11}{40}$. 10. $5500. 11. $1667.25.
12. $\$235.39\frac{1}{8}$. ($\$375\frac{5}{8} \times \frac{2}{3} \times \frac{47}{50}$). 13. 9 ac. ($\frac{2}{3}$ field in corn; $\frac{2}{9}$ in wheat; $\frac{2}{27}$ in potatoes; $\frac{1}{27}$ in beans.)

Exercise 57. Page 53

1. $\frac{1}{4}$. 2. $23\frac{1}{3}$. 3. $3\frac{7}{8}$. 4. $2\frac{13}{28}$.
5. 6. 6. 0. 7. $\frac{77}{144}$. 8. $\frac{139}{300}$.
9. 19. 10. 7. 11. $\frac{3}{4}$. 12. $1\frac{23}{30}$.

Exercise 58. Page 54 — Oral

Exercise 59. Page 54

1. 14. 2. 21. 3. $34\frac{2}{7}$. 4. $11\frac{3}{7}$.
5. 3. 6. $1\frac{29}{81}$. 7. $1\frac{27}{65}$. 8. $\frac{37}{39}$.
9. $10\frac{5}{8}$. 10. $\frac{865}{1504}$. 11. $1\frac{19}{26}$. 12. $17\frac{1}{10}$.

Exercise 60. Page 54

1. $41\frac{2}{3}$. 2. 104 yd. 3. $34\frac{25}{56}$ ft. 4. $1576\frac{4}{7}$ bu.
5. $41\frac{1}{18}$ mi. 6. $5\frac{5}{9}$ ac. 7. $79\frac{5}{8}$ lb.
8. $21\frac{5}{12}$c. $= \$\frac{107.08\frac{1}{3}}{30} \times \frac{6}{5} \times \frac{1}{20}$. 9. 24 da.
10. $(\frac{4}{15} - \frac{3}{20})$ sum $= \$28$, ∴ sum $= \$240$.
11. $7218\frac{3}{4}$ lb. $= 3\frac{39}{64}$ ton. (1 cu. ft. water weighs $62\frac{1}{2}$ lbs.).
12. $25\frac{2}{15}$. (Work backwards). 13. \$33075.

Exercise 61. Page 55

1. $7\frac{7}{8}$. 2. $3\frac{3}{4}$. 3. $7\frac{41}{130}$. 4. $\frac{3}{434}$.
5. $\frac{61}{70}$. 6. $2\frac{17}{31}$. 7. 5. 8. $21\frac{4}{13}$.
9. $1\frac{17}{45}$. 10. $\frac{75}{121} = \frac{9}{4} \times \frac{10}{3} \times \frac{2}{11} \times \frac{5}{11}$.
11. $\frac{27}{484} = \frac{9}{4} \times \frac{3}{10} \times \frac{2}{11} \times \frac{5}{11}$. 12. $\frac{15}{56} = \frac{8}{7} - \frac{7}{8}$.

Exercise 62. Page 56

1. 280. 2. 140400. 3. $11\frac{1}{4}$. 4. $13\frac{1}{3}$.
5. $\$19\frac{11}{15}$. 6. $\frac{5}{8}$. 7. $\frac{3}{8}$.
8. $85\frac{1}{3} = \frac{16 \times 24 \times 32 \times 36 \times 42}{27 \times 84 \times 96}$.
9. $245 \times 245 \times 4 + 244 = 240344$. 10. $6\frac{3}{10}$.
11. A, \$775; B, \$425. (A and B together spent \$300 and then had left \$900; of this, \$600 belonged to A and \$300 to B. ∴ at first A had \$600 + \$175, and B had \$300 + \$125).
12. 384 bu. @ 77c.; 256 bu. @ 72c. (For every bu. @ 72c. he paid 3c. less than the average, and for every bu. @ 77c. he paid 2c. more than the average. ∴ he must take 2 bu. @ 72c. for every 3 bu. @ 77c. ∴ No. bu. @ 72c. $= \frac{2}{5}$ of 640).
13. 403. ($10000 \div 457 = 21\frac{403}{457}$. ∴ 457 may be subtracted 21 times from 10000, and rem. = 403).
14. Cost of 3 geese and 5 turkeys = \$14.40.
" 5 " 3 " = \$12.
" 25 " 15 " = \$60.
" 9 " 15 " = \$43.20.
∴ " 16 " — = \$16.80.
∴ " 1 goose — = \$1.05.
15. 32640 min. (22 da. 16 hr.; 1904 is leap year).

Exercise 63. Page 57

1. $\dfrac{2\frac{1}{2}}{4}$. 2. $\dfrac{6}{4\frac{2}{3}}$. 3. $\dfrac{3\frac{1}{3}}{5\frac{1}{4}}$.

6. The division of $2\frac{1}{2}$ by $3\frac{1}{3}$. (Rather the *ratio*, or *relation*, of $2\frac{1}{2}$ to $3\frac{1}{3}$).

7. $\frac{3}{4}$. 8. $1\frac{3}{5}$, i.e. $4 \div 2\frac{1}{2}$. 9. $\frac{40}{63}$, i.e. $\frac{20}{3} \div \frac{21}{2}$.

10. $5\frac{1}{4}$ and $1\frac{1}{4}$ are in their simplest forms ; $4\frac{19}{32}$.

11. $\frac{4}{13}$; $\frac{1}{2}$. 12. The value of the fraction remains unchanged.

$$\frac{1\frac{1}{2} \times 2}{2\frac{1}{2} \times 2} = \frac{3}{5}; \ \frac{2\frac{1}{3} \times 3}{3\frac{2}{3} \times 3} = \frac{7}{11}; \ \frac{9}{11}; \ \frac{21}{27} = \frac{7}{9}; \ \frac{23}{34}; \ \frac{29}{35}.$$

13. $\frac{5}{7}$; $\frac{21}{64}$; $\frac{51}{65}$; $\frac{30}{38}$; $\frac{69}{85}$; $\frac{82}{87}$.

Exercise 64. Page 58

1. 16. 2. [illegible] 3. $\frac{5}{27}$. 4. $\frac{3}{25}$. 5. $1\frac{7}{9}$.
6. $4\frac{4}{5}$. 7. [illegible] 8. $1\frac{5}{9}$. 9. 2. 10. $2\frac{3}{4}$.
11. $7\frac{1}{2}$. 12. [illegible] 13. $\frac{465}{832}$. 14. $1\frac{73}{152}$. 15. $\frac{5}{11}$.
16. $2\frac{2}{5}$. 17. $29\frac{47}{77}$. 18. 16. 19. $1\frac{14}{57}$. 20. $\frac{17}{45}$.
21. $\frac{61}{70}$. 22. $\frac{23}{65}$. 23. $\frac{13}{17}$. 24. $\frac{94}{115}$. 25. 19.
26. $1\frac{24}{67}$. 27. $\frac{1}{2}$. 28. $\frac{201}{803}$.

Exercise 65. Page 59

1. $\frac{8}{11}$. 2. $\frac{1}{3}$. 3. 18 bags.
4. $21908. 5. $15.85. 6. 24 da.
7. $\frac{2}{3}$ of $\frac{9}{16}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{13}{28}$ 8. $5.60. 9. Too large by $\frac{15}{56}$.
10. $359.45. 11. $66.04 ($66.03$\frac{107}{191}$). 12 $\frac{2}{3}$.

Exercise 66. Page 60

7. $\frac{9}{26}$. 8. $2\frac{2}{5}$ da. 9. 3 da.
10. 9 da. 11. 16 da.

Exercise 67. Page 61

1. $6\frac{2}{3}$ hr. 2. 8 da. 3. $3\frac{7}{10}$ da. 4. $13\frac{1}{2}$ da.

5. $14\frac{2}{7}$ da. (A works for 13 da. and does $\frac{13}{20}$ of the work. ∴ B does $\frac{7}{20}$ work in 5 da.). 6. 4 hr. 7. $1\frac{1}{5}$ da.

8. $13\frac{5}{7}$ da. (A can do the work in 12 da. and B in 16 da.).

9. 7 da.

10. 20 da. { 2 A, 2 B and 2 C can mow $(\frac{1}{12}+\frac{1}{15}+\frac{1}{20})$ field in 1 day.
$= \frac{1}{5}$ " 1 "
∴ A, B and C " $\frac{1}{10}$ " 1 "
B and C can mow $\frac{1}{20}$ field in 1 day.
∴ A " $(\frac{1}{10}-\frac{1}{20})$ " 1 "
$= \frac{1}{20}$ " 1 " {

11. $5\frac{115}{121}$ da.

12. 15 da. B and C can dig $\frac{1}{12}$ garden in 1 day.
B digs $\frac{1}{20}$ " 1 "
∴ C " $(\frac{1}{12}-\frac{1}{20})$ " 1 " $=\frac{1}{30}$ garden.
∴ A " $(\frac{1}{10}-\frac{1}{30})$ " 1 " $=\frac{1}{15}$ "

13. A in 36 da., B in 45 da., C in $51\frac{3}{7}$ da.
If A can do a work in 4 da., B can do it in 5 da., and both together can do it in $2\frac{2}{9}$ da.
∴ When A and B require $2\frac{2}{9}$ da., A requires 4 da. and B 5 da.
∴ " " " 20 " " 36 " " 45 "

14. 5 da. A's time alone is 20 da., and C's alone 12 da.

Exercise 68. Page 62

1. 4*s*. 2. 4*s*. 3. $\frac{4}{25}$. 4. $\frac{77}{99}, \frac{84}{99}; \frac{7}{99}$. 5. $\frac{35}{63}, \frac{30}{63}, \frac{135}{63}; \frac{5}{63}$.

6. (1) Reduce to least common denominator;
(2) Find G.C.M. of numerators;
(3) G.C.M. of the fractions $= \frac{\text{G.C.M. of numerators}}{\text{Least common denominators}}$.

7. 48*d*. = 4*s*. 8. 48*d*. = 4*s*. 9. $\frac{45}{23}$.

10. $\frac{105}{135}, \frac{70}{135}, \frac{84}{135}$. L.C.M. $= \frac{420}{135} = \frac{28}{9} = \frac{\text{L.C.M. of 7, 14, 28}}{\text{G.C.M. of 9, 27, 45}}$.

11. $\frac{60}{15}, \frac{40}{15}, \frac{36}{15}$. L.C.M. $= \frac{360}{15} = 24$.

12. (1) Reduce fractions to their lowest common denominator.
(2) Find L.C.M. of numerators.
(3) L.C.M. $= \frac{\text{L.C.M. of numerators}}{\text{Lowest common denominators}}$.
This, with reference to the original fractions,
$= \frac{\text{L.C.M. of numerators}}{\text{G.C.M. of denominators}}$.

Exercise 69. Page 63

1. $\frac{1}{120}$; 70.
2. $\frac{1}{42}$; $\frac{40}{7}$.
3. $\frac{1}{72}$; 35.
4. $\frac{5}{128}$; 30.
5. $\frac{1}{12}$; 170.
6. $\frac{3}{4}$; 63.
7. $\frac{77}{12}$ yd. = $6\frac{5}{12}$ yd.
8. G.C.M. = $\frac{3}{200}$; L.C.M. = $\frac{36}{5}$; $\frac{5}{8}$ $\frac{3}{200}$ = 480.
9. $8350\frac{1}{3}$ hr. A has made 1927 circuits; B, 1599; C, 2444.

Exercise 70. Page 64

1. 3 pk. 1 qt. $1\frac{1}{5}$ pt.
2. 213 rd. 1 yd. 2 ft. 6 in.
3. 4 yd. 2 ft. $5\frac{1}{4}$ in.
4. £1 12*s.* $10\frac{17}{28}$*d.*; £5 2*s.* $8\frac{13}{22}$*d.*
5. 4 da. 23 hr. 28 min. ($\frac{3}{5}$ wk. = 4 da. 4 hr. 48 min.; $\frac{3}{4}$ da. = 18 hr.; $\frac{2}{3}$ hr. = 40 min.).
6. 1 lb. $7\frac{5}{13}$ oz. ($\frac{33}{1100}$ cwt. = 3 lb.; $\frac{8}{13}$ of 2 lb. 8 oz. = 1 lb. $8\frac{8}{13}$ oz.).
7. 96 rd.
8. 1750 lb. = 17 cwt. 2 qr. = 17 cwt. 50 lb.
9. 88 sq. rd. 26 sq. yd. 8 sq. ft.

Exercise 71. Page 64

1. $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{10}$.
2. $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{7}$, $\frac{3}{10}$.
3. $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$.
4. $\frac{17}{36}$.
5. $\frac{1}{33}$.
6. $\frac{2050}{2400} = \frac{41}{48}$.
7. In the lowest denomination mentioned in either of the Nos.
8. (*a*) Express each No. in the same denomination; (*b*) Write in fractional form; (*c*) Reduce the fraction to its lowest terms.
9. $\frac{3}{20}$.
10. $\frac{9}{13}$.
11. 3; 2; $\frac{1}{2}$; $\frac{1}{4}$.

Exercise 72. Page 65

1. $\frac{1}{80}$. (1 oz. = 24 scruples.)
2. $\frac{1}{40000}$.
3. $\frac{1}{360}$.
4. $\frac{1}{63360}$.
5. $\frac{1}{9}$.
6. $\frac{5}{1386}$.
7. $\frac{7}{160000}$.
8. $\frac{713}{3003}$.
9. $\frac{4161}{14875}$.
10. $\frac{361}{737}$ (1 per. = 1 sq. rd.).
11. $\frac{30}{7}$ ($\frac{3}{22400}$ t. = $4\frac{2}{7}$ oz.).
12. $\frac{44}{13}$ in. ($\frac{1}{18720}$ mi. = $3\frac{5}{13}$ in.).

Exercise 73. Page 66

1. $85.75 (1 lb. avoir. = $\frac{175}{144}$ lb. troy ; 1 lb. troy = 12 oz.).
2. $12\frac{11}{12}$ lb.
3. 65.10\frac{5}{12}$ (1 lb. avoir. = 7000 grains; 1 oz. apoth. = 480 grains).

4. $3\frac{3373}{6144}$.
5. $1\frac{19}{21}$ in.
6. £155 7*s.* $2\frac{11}{20}$*d.*
7. £34 12*s.* 4*d.*
8. 5*s.* 2*d.*
9. $11\frac{71}{80}$*d.*
10. $6\frac{1}{4}$ oz.
11. \$567.52½.
12. \$$7152\frac{48}{151}$ = \$$7152.31\frac{119}{151}$.
13. \$$173.74\frac{7}{32}$.
14. 7 hr. 11 min. 8 sec.
15. \$2.10.
16. 18 yd.
17. 98 yd.
18. $30\frac{6}{7}$ yd.
19. $4166\frac{2}{3}$ yd.
20. 21c.
21. \$1108.80.
22. £$\frac{1505}{8304}$ = 3*s.* $7\frac{86}{173}$*d.*
23. \$$1736.23\frac{1}{3}$.
24. \$880.
25. $10\frac{1}{8}$.
26. \$26160. (A has $\frac{5}{12}$ estate ; B, $\frac{2}{15}$; C, $\frac{9}{20}$).
27. $38\frac{1}{2}$ lb. ($\frac{7}{8}$ cord @ \$3.60 = \$3.15 ; tea and rice = \$1.40 ; sugar = \$1.75, etc.).
28. $1031\frac{1}{4}$ tons ; \$3300.
29. \$60.
30. $22933\frac{1}{3}$ lb. (No. bu. wheat = 120 ; value = \$77.40. No. tons $=\frac{7740}{675}=\frac{172}{15}$. ∴ No. lb. = etc.).
31. $\frac{7}{8}$ ac. ($\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{15}{16}$ ac. = $\frac{3}{8}$ ac. = 60 sq. rd.; $\frac{2}{3}$ of $\frac{12}{21}$ of $\frac{63}{60}$ of 100 sq. rd. = 40 sq. rd.; $\frac{16}{17}$ of $2\frac{1}{8}$ times 605 sq. yd. = 1210 sq. yd. = 40 sq. rd.; total = 140 sq. rd.).
32. $184\frac{102}{125}$ lb. (Value of oats = \$$17.33\frac{1}{15}$; value of apples = \$$51.97\frac{1}{2}$, etc.).
33. $4\frac{2}{3}$ ft. (Circumference $=\frac{5280}{360}$ ft.; diameter $=\frac{5280}{360}\times\frac{7}{22}$ ft.).
34. \$$349.71\frac{9}{16}$. (Length of orchard = 407 ft.; width = $251\frac{5}{8}$ ft.; perimeter = $1317\frac{1}{4}$ ft.; length of ditch = $1317\frac{1}{4}+3\frac{3}{4}\times 4$ (corners) = $1332\frac{1}{4}$ ft.).
35. \$2261. (House cost $\frac{5}{19}$ money ; cattle, $\frac{6}{19}$; farm, $\frac{8}{19}$).
36. Value of house = \$1200, and of lot = \$900. (Value of house = $\frac{4}{3}$ value of lot = $\frac{4}{7}$ of \$2100).
37. $\frac{3}{20}$.
38. $\frac{3}{7}$.
39. 100 ac. (A has $\frac{3}{4}$ land ; B, $\frac{1}{5}$; C, $\frac{1}{24}$; rem. = $\frac{1}{120}$ land = $\frac{5}{6}$ ac.).
40. 15.
41. $5\frac{1}{7}$.
42. 52.
43. 20 mi. (A must gain 5 mi. on B before he overtakes him ; A gains 1 mile in every 4 miles he walks).
44. \$93.75.
45. £41 1*s.* $5\frac{1}{7}$*d.*
46. \$8.76.
47. \$1000.
48. He sold 60 ac. for $\frac{3}{5}$ of whole cost; he sold 20 ac. for \$1200. Then $\frac{3}{5}$ of cost + \$1200 = cost; $\frac{2}{5}$ cost = \$1200; cost = \$3000.
49. $\frac{5}{39}$.
50. (*a*) $7\frac{21}{32}$; (*b*) $\frac{1}{4}$. (A guinea = 21*s.*).
51. $\frac{1}{240}$.
52. 104448.
53. 552.
54. $8\frac{1}{10}$ ac.
55. $8\frac{8}{27}$ da.

56. $77\frac{1}{2}$ ft. = $25\frac{5}{6}$ yd. (Carpet running lengthwise will require 5 strips, each $15\frac{1}{2}$ ft. long).

57. $\frac{4}{25}$; $\frac{7\frac{1}{2}}{20}$; $\frac{6\frac{1}{3}}{20}$; $\frac{3}{10}$. 58. \$$37.91\frac{1}{4}$. (£1 = \$$4.86\frac{2}{3}$).

59. 4770. (Out of every 6 voters, 5 voted = 3 for one candidate and 2 for the other; ∴ the majority of the successful candidate = $\frac{1}{6}$ No. of voters).

60. 225 lb. sulphur, $337\frac{1}{2}$ lb. charcoal, $1687\frac{1}{2}$ lb. nitre.

61. \$2.20. (Value of hay = \$24.12; value of groceries = \$21.92).

62. Dress goods = \$$9.42\frac{1}{2}$; linen = \$1.35; lining = 67c.; buttons = 36c.; thread = 15c.; tweed = \$5.94; cotton = \$2.00; cloth = \$6.75; total = \$26.65.

63. Hay = \$12.60; wood = \$18.00; apples = \$11.00; flour = \$8.75; butter = \$4.90; total = \$55.25. Balance due farmer, 80c.

64. Currants = 68c.; rice = \$1.25; soap = 75c.; cotton = \$3.01; dress goods = \$4.64; thread = 20c.; syrup = 90c.; total = \$11.43. Bal. paid on Aug. 9 = \$6.43.

65. Tea = \$3.00; sugar = \$14.25; print = \$$5.17\frac{1}{2}$; syrup = \$$1.46\frac{1}{4}$; towelling = \$1.50; knives and forks = \$$1.87\frac{1}{2}$; cheese = \$4.05; lemon peel = 52c; total = \$$31.83\frac{1}{4}$. Bal. = \$21.83 (\$$21.83\frac{1}{4}$).

66. Wheat = \$$2834.22\frac{4}{5}$; peas = \$39.26; barley = \$$20.92\frac{1}{2}$; flour = \$$6.97\frac{1}{2}$; bran = \$$35.13\frac{3}{4}$; total = \$2936.53 (\$$2936.52\frac{11}{20}$).

Exercise 74. Page 73

8. .1; .01; .001; .0001; .00001; .000001.

9. The tens occupy the second place to the left of the decimal.
" tenths " " first " " right " "
" hundreds " " third " " left " "
" hundredths " " second " " right " "

11. (*a*) Hundreds, thousands, millions. (*b*) Thousandths. (*c*) Thousands occupy the second period to the left, while thousandths occupy the first period to the right of the decimal points. (*d*) Millionths. The millions period is the third period to the left of the decimal point.

12. $\frac{72}{10}$; $\frac{84}{10}$; $\frac{65}{10}$; $\frac{97}{10}$. 13. $\frac{784}{100}$; $\frac{835}{100}$; $\frac{27}{100}$; $\frac{7}{100}$.

14. $\frac{465}{1000}$; $\frac{7250}{1000}$; $\frac{75}{1000}$; $\frac{8100}{1000}$.

15. Eighty-four, and ninety-six hundredths; three hundred and sixty-four, and seventy-two thousandths; twenty-eight, and three hundred and seven thousandths; seventeen and eight thousandths.
16. .7; .08; .27; 6.3; .078; 8.29; 7.008.
17. Seven hundred and seven thousand and five; seventy thousand seven hundred, and five tenths; seven thousand and seventy, and five hundredths; seven hundred and seven, and five thousandths; seventy, and seven thousand and five ten-thousandths.
18. To affix a cipher to the right of a whole No. *increases* the value of the No. 10 times.
19. Each cipher, as in the question, *decreases* the value of the significant digit 10 times.
20. Each may be read as four tenths, since $.40 = \frac{40}{100} = \frac{4}{10} = .4$, etc.

Exercise 75. Page 75

1. Nine tenths.
2. Twenty-seven hundredths.
3. Three hundred and sixty-eight thousandths.
4. Sixty-four thousandths.
5. Four, and thirty-one hundredths.
6. Seven, and two hundred and sixteen thousandths.
7. Three, and three hundred and fourteen thousandths.
8. Five, and eight thousand one hundred and sixty-seven ten-thousandths.
9. Twenty-one, and three thousand six hundred and one ten-thousandths.
10. Seventeen, and sixty-four ten-thousandths.
11. Eighteen, and eighty-one hundred-thousandths.
12. Twenty, and one thousand four hundred and fifty-eight hundred-thousandths.
13. .8 ; 2.07 ; .009.
14. 807.094 ; 3017.0709 ; 3.001008.
15. 6.0004 ; 80.0000609 ; 101.01001.

Exercise 76. Page 75

1. Like numbers.

2. They may be written in columns, having the units in one column, the tens in another, etc.

3.
```
   3.4
  71.61
   7.984
    .689
 367.8
```

4. 65.046.

5. 600.7354.

6. 4475.105045.

7. 2.4397464.

8. 101.209.

9. 10.867.

10. 114.1377.

11. 959.0483.

12. 40.52753.

13. 15156.66886.

14. 200.1211.

15. 25.749445.

16. 227.5024.

Exercise 77. Page 76

1. 164.25 ac.

2. 663.0388.

3. 61.19 ac.

4. 8.82 t.

5. 975.875 yd.

6. 163.135 mi.

7. 150.164575.

8. 490.3013.

9. 122.002 cwt.

10. 201.9009 mi.

11. 122.625 yd.

12. 58.4905 ac.

Exercise 78. Page 77

1.
```
 84.25
  7.56
```

2.
```
 96.
  2.75
```

3. 16.1524.

4. 2.3806.

5. .43876.

6. .23296.

7. 1.8316.

8. .00521.

9. 3.9249.

10. 1.405.

11. 168.098.

12. .01.

13. .6322.

14. 8.3416.

15. 2.5527.

16. 15.809.

17. 173.03863.

Exercise 79. Page 78

1. 36.003 gr.

2. .099.

3. .146.

4. 13.75 yd.

5. 999.999999.

6. 5.564 lb.

7. 829.375 ac.

8. .012 of the ship.

9. 22.5881.

10. 28.375 yd.

11. 52.632.

Exercise 80. Page 79

1. 22.659. 2. 28.114. 3. 87.1314. 4. 6.7605.
5. 11.5493. 6. 4.724. 7. 2.002. 8. .01.
9. 131.371 ac. 10. 553.69. 11. 2.3081. 12. 199.75 mi.

Exercise 81. Page 79

1. 15, 21, 27, 36, 39. 2. 1.5, 2.1, 2.7, 3.6, 3.9.
3. .15, .21, .27, .36, .39. 4. 1.5 2.1, 2.7, 3.6, 3.9.
5. .015, .021, .027, .036, .039.
6. .00015, .00021, .00027, .00036, .00039.
7. The No. of decimal places in the product is equal to the sum of the No. of decimal places in the multiplicand and the multiplier. 8. 7, 8, .7, .8, .07, .08.
9. The decimal point is moved one place to the right.
10. 70, 80, 7, 8, .7, 800.
11. The decimal point is moved two places to the right.
12. 723, 8010, 6.4, 700.6, 502.5.
13. To multiply a No. by 10, 100, or 1000, we simply remove the decimal point one, two, or three places to the right. (Note.—25 = 25.000 ...).

Exercise 82. Page 80

1. 4; 7. 2. 15.5440 = 15.544. 3. 240.37086.
4. .0273238. 5. 5.4008. 6. 2474.11
7. .26928. 8. 9.6142. 9. .000072.
10. .310104. 11. 803.2104. 12. .040527.
13. 1.010009. 14. .015045.

Exercise 83. Page 81

1. 56.4235 mi. 2. 6.53146. 3. 773.4375 sq. yd.
4. 1.141166125. 5. $140.432. 6. 7748977.6.
7. 334141.402 sq. in. 8. 9.75 lb. 9. 334.00692 lb.
10. 117.05936022 mi. 11. 728.9271. 12. 312.275 lb.

Exercise 84. Page 82

1. Each No. to the right is ten times the No. immediately preceding it
2. To increase it 10 times, *or* to multiply it by 10. To multiply by 100. To multiply by 1000.
4. None.
5. 700; 70; 7; .07; 110; 11.
6. 2800; 28; 2.8; 2880; 288; 28.8.
7. .8; .28; .048; .642; .008; .0008.

Exercise 85. Page 82

1. 3.07.
2. 50.615625.
3. 800.
4. .006446875.
5. 1240.
6. .00075.
7. .00016125.
8. .568.
9. 20200.
10. 22600.
11. .082.
12. .83.

Exercise 86. Page 82

1. 1562.5.
2. 300 times.
3. $\frac{.1}{.001} = 100$; 50.
4. 27.16 yd.
5. $3.24484 ..., *or* $3.24.
6. 5.08.
7. 118.614 ... bu.
8. 88.8 bu. wheat.
9. 42.55 t.
10. 518.814 bu.
11. 135 bu. each. (To buy 1 bu. each requires $1.725).
12. 3.2989894 ft. (For 25 deg., increase = .0003 of length = .0009894 ft.).

Exercise 87. Page 83

1. 168000.
2. $319.375.
3. .0001.
4. 779.01 ... gal.
5. 20.057 ... oz.
6. .11. (Sum = 102.96; $\frac{102.96}{6.05} = 17$ and remainder = .11).
7. .00421 in. (64 m. = 70 yd. = 2520 in.; ∴ 1 m. = 39.375 ... in.).
8. 2000 sheep. (He lost .44 of his sheep; he had left .56 No.; .25 of .56 No. = 280, i.e., .14 of the No. = 280).
9. $10.
10. .03012.
11. 24.3385c.
12. $13.378\dot{7}$ ac. $\left(\frac{763.4 \times 763.4}{160 \times 30\frac{1}{4} \times 9} \text{ ac.}\right)$.
13. 3887.51 ... cu. in. $(1728 \times 16 \times 1000) \div (1000 \times 7.112)$.

Exercise 88. Page 85

1. $\frac{7}{10}$. 2. $\frac{9}{25}$. 3. $\frac{3}{25}$. 4. $\frac{98}{125}$. 5. $\frac{709}{1000}$.
6. $\frac{307}{5000}$. 7. $\frac{39}{5000}$. 8. $\frac{1887}{5000}$. 9. $\frac{401}{5000}$. 10. $\frac{427}{100000}$.
11. $\frac{9}{25000}$. 12. $\frac{2007}{100000}$. 13. $\frac{142433}{200000}$. 14. $\frac{3}{500000}$. 15. $\frac{7}{8000}$.
16. .8. 17. .17. 18. .27. 19. .07. 20. .136.
21. 2.07. 22. 4.16. 23. 16.126. 24. 126.367. 25. .18496.
26. 3.00007. 27. 16.00163.

Exercise 89. Page 85

4. .875. 5. .75. 6. .375. 7. .1875. 8. .28. 9. .9375.

Exercise 90 Page 85

1. .75. 2. .625. 3. .1875. 4. .225.
5. .15625. 6. .0375. 7. 1.875. 8. .06875.
9. .078125. 10. .008. 11. 6.6. 12. 24.008.
13. 3.525. 14. 46.3125. 15. 47.140625.

Exercise 91. Page 86

1. $\frac{1}{4}$, $\frac{9}{25}$, $\frac{21}{25}$, $\frac{12}{125}$, $\frac{3}{8}$. 2. .08, .006, .056, .00364.
3. 5.7, 34.08, 6.4, 8.85. 4. $\frac{19}{50}$, $\frac{11}{25}$, $\frac{11}{250}$, $\frac{7}{80}$.
5. $7\frac{11}{20}$, $4\frac{8}{25}$, $5\frac{67}{400}$, $7\frac{1}{20}$.
6. $\frac{50}{100}$, $\frac{33\frac{1}{3}}{100}$, $\frac{66\frac{2}{3}}{100}$, $\frac{75}{100}$, $\frac{12\frac{1}{2}}{100}$, $\frac{37\frac{1}{2}}{100}$, $\frac{8\frac{1}{3}}{100}$, $\frac{41\frac{2}{3}}{100}$.
7. .75, .625, .7, .484375, .85, .075, 3.484375.
8. 6 tablecloths; 2.35 yd. remain.
9. 1.234 ; 12.34 ; 123.4 ; 1234.
10. .01234; .001234; .0001234; .00001234.
11. .0166375. $(1.25 \times 11 \times 1.1 \times .0011)$.
12. .2804 ... $\left(\frac{.25 \times .4 \times 8\frac{3}{4}}{.4 \times 7.8}\right)$.

Exercise 92. Page 87

1. $.\dot{7}1428\dot{5}$. 2. $.\dot{8}$. 3. $.8\dot{3}$. 4. $.91\dot{6}$.
5. $.\dot{5}1\dot{3}$. 6. $.38\dot{6}\dot{3}$. 7. $.\dot{9}2307\dot{6}$. 8. $.96\dot{4}2857\dot{1}$.
9. $.791\dot{6}$. 10. $.8\dot{6}$.
11. $.\dot{9}41176470588235\dot{2}$. 12. $.5\dot{1}4285\dot{7}$.

Exercise 93. Page 89

1. $\frac{2}{3}$; $\frac{6}{11}$; $\frac{437}{999}$; $\frac{1}{7}$.
2. $\frac{377}{1100}$; $\frac{707}{2475}$; $\frac{1}{15}$; $\frac{23}{66}$.
3. $\frac{2094}{495}$; $\frac{9913}{4950}$; $\frac{713}{1998}$; $\frac{294}{55}$.

Exercise 94. Page 89

1. 1.7053.
2. 1.0009.
3. 1.1686.
4. .3076.
5. $.\dot{8}3032717686\dot{1}$.
6. $1.02\dot{6}92\dot{5}$.
7. $1.7972\dot{3}22\dot{4}$.
8. $1.1\dot{7}9940156177\dot{7}$.
9. $4.67\dot{4}9\dot{0}$.
10. $.\dot{8}5714\dot{2}$; $\frac{6}{7}$.

Exercise 95. Page 90

1. 8 oz.; 8 oz.
2. 250 lb.
4. 15 min.
5. 18 in.
6. 13.2 ft.
7. 12 oz.
8. 39 min.
9. 16.2 ft.
10. 36 oz.

Exercise 96. Page 91

1. 151 sq. rd.
2. 9 oz. 15 dwt. 18 gr.
3. 10.5*d.*
4. 47 min. 6 sec.
5. 11 hr. 55 min. 40.8 sec.
6. 8*s.* 9*d.*
7. 309 rd.
8. 15 cwt. 56 lb. 4 oz.
9. 12*s.* 6¾*d.*
10. 3*s.* 5½*d.*
11. 2 da. 12 hr. 55 min. 21 sec.
12. ½*d.*

Exercise 97. Page 91

1. ⅘ ft.
2. .8 ft.
3. ¼ yd.
4. .25 yd.
5. ¾ yd.; .75 yd.
6. 5*s.*
7. .125 da.
8. .008 t.
9. .6 bu.

Exercise 98. Page 92

1. £.525.
2. .282 t.
3. .78125 oz. troy.
4. .775 mi.
5. .3125 pk.
6. £9.26875.
7. 17.895 cwt.
8. 7.875 bu.
9. .625 fath.
10. .74 of 4 oz.
11. 129.78 hr.
12. .001625 t.

Exercise 99. Page 92

1. .000625 wk.
2. $.5\dot{6}28571\dot{4}$ in.
3. 5*s.* 3*d.*
4. 6.741.
5. .40156625 mi.
6. 5.7725 wk.
7. First by 158.4 yd.
8. 175.125 lb. $\left(\frac{3 \times 15 \times .934}{.24} \text{ lb.}\right)$.
9. 126 sec. (L.C.M.)
10. 32 men. $\left(\frac{28 \times 3.663 \times 8}{16.28 \times 5 \times .315}\right)$.

Exercise 100. Page 93

1. \$4404. (.575 of A's money = \$2532.30).
2. .52. $(1.00503 \times 4) \div (9 \times .859)$. 3. £10 16*s.* $9\frac{341}{456}$*d.*
4. A, $3\frac{97}{141}$ oz.; B, $9\frac{31}{141}$ oz.; C, $7\frac{13}{141}$ oz.
(A's share = .4 of B's = .4 of 1.3 of C's = .52 of C's).
5. 220.0929 ... gal. $(1.308 \times 27 \times 1728) \div 277.274$.
6. 17888.0625 sq. ft.
7. 11436 (11435.91) = No. who can *not* read; 229320 = No. who can read; 131859 $(229320 \times .575)$ = No. who can write.
8. $\frac{57}{557}$ mi. = .1023 ... mi. 9. $\frac{91}{438}$. 10. 3.26953125 t.
11. Man's share = \$109.0827 ... ; woman's share = \$62.3329 .... (If a woman receives \$1, a man receives \$1.75; 3 men and 4 women receive \$9.25, ∴ a man's share = \$576.58 × $\frac{175}{925}$).

Exercise 101. Page 94

1. .3911 ... $\left(\text{Fr.} = \frac{£8 \quad 9s.\ 3d.}{£21\ \ 12s.\ 9d.} = \frac{2031d.}{5193d.} = \frac{677}{1731} = \text{etc.}\right)$.
2. 39 mi. 3. 37678.275 ft. 4. 149.4475 sq. rd.
5. \$1278.75. (.125 = ½ of .25).
6. 259 mi. 81 rd. 3 yd. 2 ft. $5\frac{7}{25}$ in. 7. .62963 yd.
8. $\frac{40}{63}$. $\left(\frac{114}{399} + \frac{12}{28} - \frac{674}{3033} = \frac{2}{7} + \frac{3}{7} - \frac{2}{9}\right)$. 9. 605.0745 lb.
10. \$1.80. (.275 price = $\frac{2}{9}$ price = 9½c.).

Exercise 102. Page 94

1. Thirty thousand three hundred and three, and three hundred and three ten-thousandths.
2. 7000.007. 3. \$227.50.
4. 9012½ grs. (1 gal. water weighs 10 lb.; ∴ 1 gal. milk weighs 10.3 lb.; ∴ 1 pt. milk weighs $\frac{10.3}{8}$ lb. = $\frac{10.3}{8} \times 7000$ grs.).
5. .0027855 .... 6. \$262.50. (No. of 5c. pieces = $12 \times 7000 \div 16$).
7. Green, 12 oz.; black, 1 lb. 4 oz.
8. Turkeys, 85c. each; geese, 60c. each. (If the turkeys were the same price as the geese, the woman would have received \$2 less, i.e., 8 @ 25c.; ∴ s.p. of 16 geese = \$9.60).

9. John, \$200; James, \$400; Henry, \$1200.
10. \$70. (No. strips $= 17\frac{2}{3} \div 2\frac{1}{4} = 8$; length of each strip = 21 ft. = 7 yd.).
11. A, \$250; B, \$160; C, \$110.
12. Man, \$1.50; boy, 75c. (Weekly wages of 5 men = \$45).
13. Men, \$56.70; boys, \$25.90. (27 men receive as much as 81 boys).
14. $133\frac{1}{3}$ cwt.; 100 cwt. nitre; $13\frac{1}{3}$ cwt. sulphur. ($\frac{3}{20}$ total wt. = 20 cwt.).
15. \$8. (Wages for 12 mo. = \$160 + 10 sheep; wages for 7 mo. = \$60 + 10 sheep; ∴ wages for 5 mo. = \$100; wages for 12 mo. = \$240 = \$160 + 10 sheep; ∴ value of 10 sheep = \$80).
16. $3\frac{13}{14}$ mi. $\{(160 \times 30\frac{1}{4} \times 9 \times 5) \div (14 \times \frac{3}{4} \times 3 \times 1760)\}$ mi.
17. A, $\$266\frac{2}{3}$; B, $\$933\frac{1}{3}$.
18. \$24.81; \$33.08. (Ratio of 3 to 4).
19. \$1. {2 men get as much as 5 boys; 6 men get as much as 15 boys; 12 women get as much as 18 boys; ∴ (15 + 18 + 15) boys would get \$19.20; 1 boy would get 40c.; 1 man would get 40c. $\times \frac{5}{2}$}.
20. 109 ft. $4 \times 6 - 91 \times 5$).
21. 75c. $\$1.20 \times 6 - \frac{\$(1.80 + 2.40)}{4}$.
22. \$32.20; $57\frac{1}{2}$c.
23. 23 bu.; \$598.
24. 1716 yd.; $71\frac{1}{2}$ yd.
25. 30c. $\left(\frac{23 \times 10 - 20 \times 7}{3}\right)$.
26. $37\frac{2}{3}$ lb.; $8\frac{716}{935}$c.
27. \$5000; \$25.
28. $18.33\dot{8} = 18\frac{61}{180}$.
29. 26.
30. 510 lb. $(550 \times 6 + 462 \times 5) \div 11$.
31. $25\frac{5}{9}$ gal. $\left(\frac{4.60 \times 92}{3.60} - 92\right)$.
32. 4928 cu. ft. (Length of wall = 308 ft).
33. $148\frac{1}{2}$ mi. (No. furrows = 792).
34. $307\frac{5}{9}$ sq. yd.
35. 14580. (6 in. to the weather require 45 courses for each side. Shingles 4 in. wide require 162 to the course; ∴ $162 \times 45 \times 2$).
36. \$30.
37. \$448.80.
38. $170\frac{29}{33}$ yd.
39. \$50. (No allowance for doors or windows).

Exercise 103. Page 99

10. $\frac{1}{20}$, $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$. 11. 30 in.
12. 7%, 12%, 25%, 30%. 13. 10%, 5%, 4%, $12\frac{1}{2}$%.
14. $\frac{50}{100}$, 50%, $\frac{25}{100}$, 25%; $\frac{20}{100}$, 20%; $\frac{12\frac{1}{2}}{100}$, $12\frac{1}{2}$%; $\frac{6\frac{1}{4}}{100}$, $6\frac{1}{4}$%; $\frac{10}{100}$, 10%; $\frac{60}{100}$, 60%; $\frac{75}{100}$, 75%; $\frac{33\frac{1}{3}}{100}$, $33\frac{1}{3}$%; $\frac{66\frac{2}{3}}{100}$, $66\frac{2}{3}$%.
15. 50%, 25%, 20%, 25%. 16. 100%.

Exercise 104. Page 100

1. $\frac{7\frac{1}{2}}{100}$. 2. $\frac{6\frac{1}{4}}{100}$. 3. $\frac{33\frac{1}{3}}{100}$. 4. $\frac{12\frac{1}{2}}{100}$. 5. $\frac{37\frac{1}{2}}{100}$.
6. $\frac{87\frac{1}{2}}{100}$. 7. $5\frac{1}{2}$%. 8. $7\frac{1}{2}$%. 9. 25%. 10. $25\frac{1}{4}$%.
11. $3\frac{3}{10}$%. 12. 12500%. 13. 1/[illegible]. 14. $\frac{23}{200}$. 15. $\frac{1}{8}$.
16. $\frac{1}{3}$. 17. $\frac{77}{200}$. 18. $\frac{7}{8}$. 19. 25%. 20. 20%.
21. 75%. 22. $33\frac{1}{3}$%. 23. 40%. 24. $37\frac{1}{2}$%.

Exercise 105. Page 100

1. 64 ; 72 ; 100. 2. 88 ; 104 ; 120. 3. 128 ; 144 ; 160.
4. 72. 5. $15. 6. 23 sheep.
7. 45 ft. 8. $25. 9. 15 mi.
10. $10.50. 11. 14 men. 12. 45.
13. 6 men. 14. 48 sheep. 15. $280.

Exercise 106. Page 101

2. 6%. 3. $\frac{7}{25}$; 28%. 4. $\frac{13}{200}$; $6\frac{1}{2}$%.
5. 50%, 25%, 20%, 75%, $16\frac{2}{3}$%, 40%. 6. 32%.
7. 40%. 8. 75%. 9. 60%; $36\frac{4}{11}$%.
10. $8\frac{1}{3}$%. 11. 40%. 12. $66\frac{2}{3}$%. 13. $19\frac{731}{1051}$%.

Exercise 107. Page 101

1. 1100. 2. $1200. 3. 357 ac. 4. $208\frac{1}{3}$ ac.
5. $4 each. 6. $100. 7. $2050. 8. 20 lb.
9. $7280. 10. 605. 11. $4800. 12. 2880.

Exercise 108. Page 102

1. 110 men, 88 women, 66 boys. (If No. of boys be 3, No. women is 4, and No. men is 5). 2. 3600 ; 4185.
3. 13310. 4. $12937.50. 5. $4076.163 ; $\frac{421}{810}$.
6. $1620. 7. 600 bu.; 720 bu. 8. $20.
9. $4700. 10. B, $\frac{5}{27}$; C, $\frac{5}{23}$. 11. $\frac{7}{60}=11\frac{2}{3}\%$.
12. $6\frac{1}{4}\%$. $(380 - 340) = twice the gain when goods are sold for $340; ∴ cost of goods = $320. 13. 24 yr.

Exercise 109. Page 103

1. $480; $1680. 2. $1183.50. 3. $567; 1.13\frac{3}{8}$.
4. $8\frac{1}{4}$c. 5. $720. 6. $3726.
7. $9806.28. 8. $699.30. 9. $420.
10. $240. 11. 12.
12. 3808.86\frac{2}{3}$. (3% cost = $117.80).
13. (*a*) $12, 50% ; (*b*) $105, 25% ; (*c*) $60, 25%.; (*d*) $160, $40; (*e*) $1450, 20% ; (*f*) $1860, $620 ; (*g*) $1860, $124.
14. (*a*) $8, $14\frac{2}{7}\%$; (*b*) $8, 25% ; (*c*) $80, $16 ; (*d*) $160, $40 ; (*e*) $200, $17\frac{1}{2}\%$.

Exercise 110. Page 105

1. $225. 2. $400. 3. $108. 4. 25%, or $125.
5. $2.50. 6. $150. 7. $45. 8. $57.

Exercise 111. Page 106

1. $405. 2. $155. 3. $584.25.
4. $1114.92. 5. $380. 6. $301.53.
7. $12.75. 8. $137.88. 9. $37\frac{1}{2}\%$.
10. $37\frac{1}{2}\%$. 11. $500. (Cost = $216 ; gain = 25% of $216 = $54; s. p. = $270. If list price = $100, 1st dis. = $25; 1st reduced price = $75; 2nd dis. = $15; 2nd reduced price = $60 ; 3rd dis. = $6 ; 3rd reduced price = $54 ; ∴ s. p. = $\frac{54}{100}$ list price = $270).

Exercise 112. Page 107

1. $28\frac{3}{4}$%.   2. $35\frac{7}{8}$%.   3. 28%.
4. 25%.   5. $289.80; $19\frac{1}{2}$%.   6. $250.
7. $400.   8. First.   9. $49\frac{1}{3}$%.
10. $855.

Exercise 113. Page 108

1. $4, or 20%.   2. $4, or 20%.   7. $110   8. 50%.
9. 20%.   10. 15%.

Exercise 114. Page 109

1. 25%.   2. 20%.   3. 4%.
4. $11\frac{1}{9}$% loss.   5. $62\frac{1}{2}$%.   6. 44%.
7. 21%.   8. 125%.   9. 25%.
10. $58\frac{1}{3}$%.   Value = $140.

Exercise 115. Page 109

1. $45.80.   2. $1.45.   3. $5.07.
4. $21\frac{1}{2}$%.   5. $6\frac{1}{4}$%.   6. $3125.
7. $9.90.   8. $320.   9. 11c.
10. Merchant; 50c. (Cost of wheat to the farmer = $22.50; gain = $4.50. Cost of goods to merchant = $20; gain = $5).
11. $250 loss. (Cost of 1st farm = $2500; cost of 2nd = $3750).

Exercise 116. Page 111

1. $20.   2. $237.50.   3. $1200.
4. Cost to purchaser = $8500; sum received by seller = $8160.

Exercise 117. Page 111

1. $14.40.   2. $15.90.   3. $10.
4. $30.   5. $247.   6. $112.
7. $37.50.   8. 13.12\frac{1}{2}$.
9. Com., $117; sum paid to employer, $2483.
10. $1086.75   11. 5659.87\frac{1}{2}$; com., 145.12\frac{1}{2}$.
12. $4384.25.

Exercise 118. Page 112

1. 5%. 2. 3$\frac{1}{4}$%. 3. 2%. 4. 5$\frac{3}{4}$%.
5. 2$\frac{1}{3}$%. 6. 2$\frac{114}{351}$%. 7. 2$\frac{1}{2}$%. 8. 13$\frac{119}{2737}$%.
9. 2%. Net proceeds of sale = \$680 − \$136 (freight) − \$34 (com.) = \$510, *or* 75% of \$680.
10. 4$\frac{1}{2}$%. Com. on \$325 = \$14.62$\frac{1}{2}$.

Exercise 119. Page 113

1. \$610. 2. \$1636. 3. \$448. 4. \$1400.
5. \$16.45. 6. \$820.30. 7. \$2500.
8. 65c. (First cost = \$62 × $\frac{100}{2\frac{1}{2}}$ = \$2480. Com. + freight = \$120. Total cost = \$2600).
9. 680 bbls. (Com. for buying 1 bbl. = 2% of \$5 = 10c. Total cost of 1 bbl. = \$5.10; ∴ No. bbls. = $\frac{3468}{5.10}$). 10. 13333$\frac{1}{3}$ bu.
11. 30%. Net proceeds = \$1750 − \$(87.50 + 25 + 12.50) = \$1625.

Exercise 120. Page 116

1. \$25. 2. \$100. 3. \$48. 4. \$4000.

Exercise 121. Page 116

1. \$18. 2. \$11.20. 3. \$15.20. 4. \$11.25.
5. \$100. 6. \$110.40. 7. \$166.25. 8. \$65.20.
9. \$47.10. 10. \$30.75. 11. \$69.60. 12. \$171.75.
13. \$487.50.

Exercise 122. Page 116

1. 1$\frac{3}{4}$%. 2. $\frac{3}{4}$%. 3. $\frac{3}{5}$%.
4. 1$\frac{547}{4375}$%. 5. 1$\frac{1}{3}$%. 6. 3$\frac{1}{3}$%.
7. 1$\frac{1}{8}$%. Prem. = \$(6000 − 5930 − 2.50) = \$67.50.
8. $\frac{5}{8}$%. Policy = \$288000 ; prem. = \$1800.
9. \$27510. Policy = \$28000 ; prem. = \$490. 10. \$3739.20.

Exercise 123. Page 117

1. \$70000. 2. \$46000. 3. \$7792. 4. \$12000.
5. \$2500. 6. \$29730. Policy = \$270 × $\frac{1000}{9}$ = \$30000.
7. \$9200. 8. \$30187.50.

Exercise 124. Page 120

1. $100. 2. $120. 3. 1½%, *or* 15 mills ; $113.85.
4. $43.20. 5. $5280. 6. $110.88. 7. $612.50.
8. $7.12½. 9. 15 mills, *or* 1½%. 10. $164.06¼.
11. (*a*) $54 ; (*b*) 2 mills. 12. 3¼ mills.

Exercise 125. Page 122

1. $675.40. 2. $655.35. (No. ac. *assessable* land = 15420).
3. $52. 4. $409.60. (There are 4 quarter-sections to a sq. mile; ∴ 64 quarter-sections).
5. 6c. per ac. (No. ac. assessable = 640 × 12 − 480).
6. 7c. per ac. (Tax for 1907 = $518.40 ; for 1908 = $806.40).
7. 640 ac. 8. 4 mills. 9. $25.50.
10. $12. (He pays taxes on $800).
11. 16 mills. (Taxes on an assessment of $1800 = $28.80).
12. $2400. ($\frac{16}{1000}$ of the *taxable* income = $25.60).
13. 20 mills ; $50000. (Taxes on $15000 = $300).

Exercise 126. Page 123

1. $8400. 2. $330000. 3. $600000.
4. $304000. ($\frac{95}{100}$ total taxes = $3610). 5. $4218.75.
6. $3250. $\left(\frac{14\frac{2}{5}}{1000}\text{ total assessment} = \$109.08\right)$. 7. $24400.
8. 2000 lb. (Sp. duty on 100 lb. = $2 ; *ad val.* duty on 100 lb. = 10% of $4 = 40c.; total duty on 100 lb. = $2.40).
9. $3720000. (97½% total tax = $18135).
10. 207360 ac.; $18662.40.

Exercise 127. Page 124

1. $8000. (95% of 97½% of sum levied = $7410).
2. $3000. 3. $607.95.
4. $148.932. (Duty calculated on 98½% of invoice price).
5. 280 gal. {*Ad val.* duty per gal. = 30% of $1.50 = 45c.; total duty per gal. = $(2.40 + .45) = $2.85}.
6. $1764. (25% invoice pr. = $367.50 ; gain = 20% invoice pr.).

7. 35%. (Duty on \$875 worth of goods = \$306.25).

8. \$2450. $\left(\frac{983\frac{3}{5}}{1000}\right.$ *taxable* income = \$1721.30$\left.\right)$. 9. \$2360000.

10. \$4871.25. 11. (*b*) No. of sections = 324 ; No. of acres = 207360. (*c*) Assessment = \$3317760.

12. (*a*)

Date.	Principal.	Interest.	Total.
Dec. 15, 1913	\$1200.	\$720.	\$1920.
" 1914	1200.	648.	1848.
" 1915	1200.	576.	1776.
" 1916	1200.	504.	1704.
" 1917	1200.	432.	1632.
" 1918	1200.	360.	1560.
" 1919	1200.	288.	1488.
" 1920	1200.	216.	1416.
" 1921	1200.	144.	1344.
" 1922	1200.	72.	1272.
	\$12000.	\$3960.	\$15960.

(*b*) 3 mills. (Sum to be raised = \$1848 + \$5616.96).
(*c*) \$71.68. (Assessment = \$16 × 640. Rate = 7 mills).

Exercise 129. Page 127

1. \$30. 2. \$38.50. 3. \$52.50. 4. \$236.412.
5. \$233.3295. 6. \$328.95. 7. \$568.05. 8. \$451.50.
9. \$236.64. 10. \$314.60. 11. \$729.90125.
12. \$63.21: 13. \$311.64. 14. \$96.738.

Exercise 130. Page 128

1. \$520. 2. \$882. 3. \$1887.60. 4. \$934.92.
5. \$784.665. 6. \$954.56. 7. \$2002.65. 8. \$14.958.
9. \$215.753; (175 da.) 10. \$16.856: (215 da.)

Exercise 131. Page 129

1. 8%. 2. $7\frac{1}{2}$%. 3. 6%. 4. 5%.
5. $5\frac{2}{5}$%. 6. 9%. 7. 7%. 8. $4\frac{1}{2}$%.
9. $2\frac{1}{50}$%. 10. \$1017.50. (Int. = \$17.50; 7%).
11. 8%. (If prin. = \$100, int. for 25 yr. = \$200; int. 1 yr. = \$8).

Exercise 132. Page 130

1. 3 yr. 2. $3\frac{9}{13}$. 3. 2 yr. 3 mo. = $2\frac{1}{4}$ yr.
4. 5 mo. 5. $\frac{47}{73}$ yr. = 235 da.
6. Dec. 2. (175 da. after June 10). 7. 3 yr.
8. Oct. 4, 1900. ($4\frac{3}{8}$ yr. = 4 yr. 137 da.).
9. $14\frac{2}{7}$ yr. (Int. on \$100 = \$7 for 1 yr. = \$100 in $\frac{100}{7}$ yr.).
10. $16\frac{2}{3}$ yr.

Exercise 133. Page 130

1. \$760. 2. \$846. 3. \$3500. 4. \$2000.
5. \$9000. 6. \$32500. 7. \$360. 8. \$655.
9. \$750. 10. £491 13*s.* 4*d.* 11. \$1825.
12. \$240. (Prin. + int. for $3\frac{3}{4}$ yr. = \$312; prin. + int. for $3\frac{1}{2}$ yr. = \$307.20; int. for $\frac{1}{4}$ yr. = \$4.80; int. for $3\frac{1}{2}$ yr. = \$67.20).

Exercise 134. Page 132

1. \$19.118 ... (1 yr. 57 da.) 2. \$9.1613 ... (1 yr. 202 da.).
3. \$700. 4. \$15.6722 ... (246 da.).
5. \$392.40. 6. \$17.40. (Int. = \$.896 ... ; 305 da.).
7. Gain on Jan. 8 = \$35.20. (Int. = \$4.80; 146 da.).
8. \$81.668 ... (145 da.). 9. \$881.313 ... (218 da.).
10. \$563.50. (Int. on \$1000 for $1\frac{1}{2}$ yr. @ 6% = \$90; bal. due on July 1, 1911 = \$490; int. on \$490 for $2\frac{1}{2}$ yr. @ 6% = \$73.50).

Exercise 135. Page 132

1. (*a*) May 15, 1909, 1910, 1911, 1912.
   (*b*) \$155 ; \$147.50 ; \$140 ; \$132.50. (*c*) \$575.
   (*b*) First year int. is on \$500; the second year on \$375, etc.
2. (*a*) 10 yr. (*b*) \$56; \$196; \$50.40; \$190.40; \$44.80; \$184.80; \$39.20; \$179.20; \$33.60; \$173.60; \$28; \$168; \$22.40; \$162.40; \$16.80; \$156.80; \$11.20; \$151.20; \$5.60; \$145.60. (\$56 = int. on \$1400 for 1st $\frac{1}{2}$ yr.; \$196 = int. for $\frac{1}{2}$ yr. + \$140; \$50.40 = int. on \$1260 for 2nd $\frac{1}{2}$ yr.; \$190.40 = int. for $\frac{1}{2}$ yr. + \$140, etc.). (*c*) \$616.
3. (*a*) April 1, 1913-4-5-6-7-8-9, 1920-1-2.
   (*b*) \$186; \$179.40; \$172.80; \$166.20; \$159.60; \$153; \$146.40; \$139.80; \$133.20; \$126.60. (*c*) \$363.

4. (*a*) $222; $213; $204; $195; $186; $177; $168; $159.
(*b*) $324.

5. (*a*)

Date.	Principal	Interest.	Total.
Oct. 1, 1913	$100.	$30.	$130.
“ 1914	100.	54.	154.
“ 1915	100.	48.	148.
“ 1916	100.	42.	142.
“ 1917	100.	36.	136.
“ 1918	100.	30.	130.
“ 1919	100.	24.	124.
“ 1920	100.	18.	118.
“ 1921	100.	12.	112.
“ 1922	100.	6.	106.
	$1000.	$300.	$1300.

(*b*) $1300.

6.

Date.	Principal.	Interest.	Total.
Mar. 15, 1914	$800.	$720.	$1520.
“ 1915	800.	672.	1472.
“ 1916	800.	624.	1424.
“ 1917	800.	576.	1376.
“ 1918	800.	528.	1328.
“ 1919	800.	480.	1280.
“ 1920	800.	432.	1232.
“ 1921	800.	384.	1184.
“ 1922	800.	336.	1136.
“ 1923	800.	288.	1088.
“ 1924	800.	240.	1040.
“ 1925	800.	192.	992.
“ 1926	800.	144.	944.
“ 1927	800.	96.	896.
“ 1928	800.	48.	848.
	$12000.	$5760.	$17760.

7. (*a*) Ap. 15, 1903. (*b*) Amount of last payment = $1060.

Exercise 136. Page 137

1. Amount of deposit = \$200.07. 3. \$332.46.
4. Deposit = \$407.38. 6. \$848.37.

Exercise 138. Page 138

1. (1) July 24, 1913. (2) 80 da. (3) \$493.42.
2. (1) Jan. 18, 1914. (2) 125 da. (3) \$390.41.
3. (1) Ap. 28, 1913. (2) 73 da. (3) \$647.145, *or* \$647.15.
4. (1) Sep. 14, 1913. (2) 45 da. (3) \$507.22.
5. (1) Ap. 22, 1913. (2) 55 da. (3) \$115.39.
7. (1) Due date = Dec. 11. (2) Term of dis. = 215 da. (3) Dis. = \$584 × $\frac{5}{100}$ × $\frac{215}{365}$ = \$17.20; proceeds = \$566.80.
8. \$3.60. 9. (*a*) Sep. 30, 1914. (*b*) Oct. 3, 1914. (*c*) Face value + int. for 187 da. = \$800 + \$30.74 = \$830.74. (*d*) Dis. = \$830.74 × $\frac{9}{100}$ × $\frac{187}{365}$ = \$38.31; proceeds = \$792.43.
10. Note matures, Feb. 15; discounted, Jan. 1; term of dis. = 45 da. Dis. = \$1200 × $\frac{7}{100}$ × $\frac{45}{365}$ = \$10.36.

Example 8. Page 139

Dis. for 33 days @ 6% = $\frac{33}{365}$ of $\frac{6}{100}$ of face value of note
= $\frac{99}{18250}$ face value;
∴ Proceeds = Face − $\frac{99}{18250}$ face = $\frac{18151}{18250}$ face = \$556.92;
∴ Face value = \$556.92 × $\frac{18250}{18151}$ = \$559.96.

Exercise 139. Page 139

1. \$74.58. (Dis. = $\frac{93}{365}$ of $\frac{7}{100}$ face = $\frac{651}{36500}$ face; proceeds = $\frac{35849}{36500}$ face = \$73.25; ∴ Face = etc.).
2. \$714.29. (Dis. = $\frac{3}{12}$ of $\frac{8}{100}$ face = $\frac{1}{50}$ face; proceeds = $\frac{49}{50}$ face = \$700).
3. Due date = Jan. 4, 1915; term of int. = 1 yr. 3 da.; int. = \$40.33; maturity value = \$540.33; date of dis. = Sep. 4, 1914; term of dis. = 122 da.; dis. = \$540.33 × $\frac{9}{100}$ × $\frac{122}{365}$ = \$16.25; proceeds = \$524.08.

4. $1723.75. (Due, Dec. 13. Term of dis. = 73 da. Dis. = $1750 × $\frac{15}{200}$ × $\frac{73}{365}$ = $26.25).
5. $184. (Dis. = $\frac{3}{12}$ × $\frac{15}{200}$ face = $\frac{3}{160}$ face = $3.45).
6. $336. (Dis. = $\frac{3}{12}$ × $\frac{25}{400}$ face = $\frac{1}{64}$ face = $5.25).

Exercise 140. Page 141

3. $246.
4. 22$\frac{13}{16}$%. (Dis. on $1200 for 40 da. = $30; dis. on $100 for 1 yr. = $30 × $\frac{1}{12}$ × $\frac{365}{40}$ = 22\frac{13}{16}$. A note is *due* only when it is *legally* due)
5. $584. (Dis. = $\frac{90}{365}$ × $\frac{15}{200}$ face = $\frac{27}{1460}$ face: proceeds = $\frac{1433}{1460}$ face = $573.20).
6. $\frac{2}{5}$ yr. *or* 146 da. (Dis. for 1 yr. = $25).
7. (1) Proceeds = $572.32. (Term of dis. = 73 da.).
   (2) Rate = 8%. (Term of dis. = 73 da.).
   (3) Proceeds = $695.35. Date of dis. = Feb. 7.
   (4) Date of note = July 26. (Dis. = $35.75; dis. for 1 yr. = $137.3715; term of dis. = $\frac{35.75}{137.3715}$ yr. = 95 da.; due date = 95 da. *after* Aug. 26 = Nov. 29; date of note = 4 mo. *before* Nov. 26: Nov. 26 to Nov. 29 being the *days of grace*).
   (5) April 17. (Term of dis. = 80 da.; due date = July 20).

Exercise 141. Page 142

1. $102.50; $1102.50.
2. $477.54; $2977.54.
3. $312.16; $2812.16.
4. C. int. = $191.02; S. int. = $180; diff. = $11.02.
5. C. int. = $153.02; S. int. = $150; diff. = $3.02.
6. C. int. = $206.08; S. int. = $200; diff. = $6.08.
7. $124.16.
8. $6.90.
9. $466.56. {1.16 sum = $464; sum = $400; ∴ $400 $(1.08)^2$}.
10. $129.86. (.24 sum = $120).
11. $541.22, i.e., $500 $(1.02)^4$.

Exercise 142. Page 143

1. \$6000. 2. 160 ft. 3. 3960.
4. \$487.50. 5. \$3816; \$954. 6. \$1622250.
7. 25%; $21\frac{3}{7}$%; 20%. 8. $\frac{43}{200}$; \$22000. 9. \$14400; \$9000.
10. $3277\frac{7}{20}$. 11. 30 wk. 12. 1.9453125.
13. $\frac{404}{1100 \times 4} = \frac{101}{1100}$. 14. 450 ft. board measure. 15. $3\frac{11}{14}$.
16. 132 yd. (Carpet running lengthwise: 11 strips of 12 yd. each)
17. \$1566. (Bank dis.). 18. $63\frac{7}{11}$%.
19. \$230.85. 20. $1\frac{1}{11}$ hr. 21. .0062112.
22. $\frac{14}{15}$. 23. \$63; \$105.
24. \$5050.80. (1 ac. = 10 sq. ch.). 25. \$99.74.
26. \$450.
27. $\$640\frac{99}{100}$. Regina, July 31, 1914.

Four months after date I promise to pay Fred. Thomas, or order, the sum of six hundred and forty dollars, with interest at 8 per cent. per annum. Value received. A. Brown.

28. \$4800. 29. 85%. 30. 88c. per lb.
31. \$6.30. 32. 3 hr. (1 cu. ft. = $6\frac{1}{4}$ gal.).
33. \$80; $\$93\frac{1}{3}$. (Farms cost \$12000 and \$14000). 34. \$60.
35. $924 = 2^2 3 \times 7 \times 11$; $2520 = 2^3 \times 3^2 \times 5 \times 7$; G.C.M. $= 2^2 \times 3 \times 7 = 84$. All the factors of 84 are common factors of 924 and 2520; ∴ 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84 are common factors.
36. .765625. 37. $1\frac{1}{4}$% for 3 yr. 38. \$4000.
39. \$1000. 40. 901. 41. 34.28 mi. per hr.
42. \$1320; \$1760. 43. Turkey, \$2.65; goose, \$1.35.
44. \$3.75.
45. 33 ac. 95 sq. rd. ($2\frac{1}{2}$ in. by $\frac{7}{8}$ in. represent 100 rd. by 35 rd.; $1\frac{7}{8}$ in. by $\frac{5}{8}$ in. represent 75 rd. by 25 rd.). 46. 60%.
47. \$3.60. (Cost = \$2.70; gain = $\frac{1}{5}$ of \$2.70; s.p. = \$3.24 = $\frac{9}{10}$ m.p.).
48. Amount of bill = \$5.96 ($\$5.96\frac{1}{4}$); dis. = 30c.; am't p'd = \$5.66.
49. 30.98. 50. 20 each. (If one of each, value = 3*s*. $7\frac{1}{4}$*d*.; ∴ No. of each $= \frac{\pounds 3\ 12s.\ 1d.}{3s.\ 7\frac{1}{4}d.} = 20$).

51. £1 3*s.* 7½*d.*; £1 15*s.* 5¼*d.* 52. $41.10. (Surface = 34¼ sq. ft.).
53. $5\frac{3}{5}$ ft. must be cut off the 24 ft. board. 54. $6.40.
55. A, $800; B, $1400; C, $700. (A's share = $1\frac{1}{7}$ times C's; B's = twice C's; $4\frac{1}{7}$ times C's share = $2900). 56. $62.50; $67.50.
57. 128 lb. (A diff. of 4c. a bag gives a total diff. of $50; ∴ No. of bags = 1250). 58. 80 ft.
59. 12 sec. (Train goes 176 yd. in order to *clear* 100 yd.).
60. $46.30. (Value of a turkey is $1.30, and of a goose 70c.).
61. A, $10; B, $8.40; C, $6.60. (B's share = C's share + $1.80; A's share = C's share + $3.40; ∴ 3 times C's share + $5.20 = $25). 62. $32. 63. 96c.
64. 328. (4 units + 5 sixes + 7 forty-twos). 65. 33¾ gal.
66. A, 30c.; B, 36c.; C, 40c.
67. $55. No. sold at 1st = $\frac{1120}{40}$ = 28; loss on 28 = $1.40, or $5 each; cost of each = $45; No. bought = $\frac{2790}{45}$ = 62; No. left = 34; s.p. of 34 = $(2790 + 200 − 1120) = $1870; s.p. = $55 each.
68. 18. (One at each end). 69. $8.40. (No. of posts = 24).
70. $6. (No. of posts = 30).
71. $40. (16 posts; 900 ft. lumber; 200 ft. scantling).
72. 41.02\frac{14}{15}$, *or* $41.03. (24 posts; 920 ft. lumber; 245⅓ ft scantling). 73. 10779$\frac{19}{49}$ bu. (Wheat, $5157.49; oats, $50.50; barley, $73.91). 74. 97½%. (Possible day's attendance = 1180; actual day's attendance = 1150½).
75. First. $\left(\frac{10}{9.009} = 1.11000111\ldots;\ \therefore \text{Error} = .00000111\ldots;\ \frac{10}{1.11} = 9.009009\ldots;\ \therefore \text{Error} = .000009\ldots\right)$.
76. $252.16. (1st com. = 2½% of $6398.56 = $159.96; balance = $6238.60; 2nd com. = $6238.60 × $\frac{1\frac{1}{2}}{101\frac{1}{2}}$ = $92.20).
77. 7 oxen, 14 cows, 42 sheep.
78. Outer circumference = 44 in.; outer dia. = 14 in.; inner dia. = 7 in.; inner circumference = 22 in. Area to be painted = $\left(\frac{44+22}{12} \times 21\right)$ sq. ft. Cost = $17.32½, *or* $17.33.
79. 398 mi. (12 hr. 5 min. from Brandon to Moose Jaw).
80. 810178.991919. (Other No. = 900.1; sum = 901.109; diff. = 899.091).

81. $2\frac{7}{9}$% gain. {Cost at Moosomin = \$2880 ; weight = 480000 lb.; freight = \$1920 ; total cost in Revelstoke = \$(2880 + 1920 + 384) = \$5184. Loss in wt. = $7\frac{1}{2}$% of 480000 lb.; No. lb. sold = 480000 − 36000 = 444000; proceeds = \$24 × $\frac{444000}{2000}$ = \$5328 ; gain = \$144 ; gain % = $\frac{144}{5184}$ × 100 = $2\frac{2}{9}$. *Note.*—If freight be paid on Revelstoke weight, then freight = \$1776. Gain % = $5\frac{5}{7}$.}

82. \$461; \$365. (1st works 461 hr. and 2nd 365 hr.).

83. \$6.24 (\$6.24078). (Cost per m. = 7 fr. ; 5 yd. = 5 × 36 × .0254 m.; cost = 7 × 5 × 36 × .0254 fr. = 19.5 × 7 × 5 × 36 × .0254c. = \$6.24).

84. (*a*) \$161. (Perimeter = 560 ft.; 4620 ft. lumber ; $746\frac{2}{3}$ ft. scantling). (*b*) Area = 9856 sq. ft. ; circumference = 308 ft. (Width of corral = 120 ft.; dia. of circular pen = 112 ft.; circumference = $\frac{22}{7}$ × 112 ft. Area = $\frac{22}{7}$ × 56 × 56 sq. ft.). (*c*) 200 ft.

85. (*a*) 665338617 ; 472300192081. (*b*) .0056 ; .7.

86. A's loss = \$1440 ; B's loss = \$2385.

$$\left.\begin{array}{l}\text{(A has \$2500 in trade for 9 mo.} = \$22500 \text{ for 1 mo.}\\ \text{`` \quad 1700 \quad `` \quad `` \quad 15 `` } = \ 25500 \text{ `` 1 ``}\end{array}\right\} = \$48000 \text{ for 1 mo.}$$

$$\left.\begin{array}{l}\text{B has \$3000 in trade for 9 mo.} = \$27000 \text{ `` 1 ``}\\ \text{`` \quad 3500 \quad `` \quad `` \quad 15 `` } = \ 52500 \text{ `` 1 ``}\end{array}\right\} = \$79500 \text{ for 1 mo.}$$

(Loss is shared in proportion of 480 to 795).

Exercise 144. Page 151

1. 72 ; 126.
2. 55 ; 66 ; 77.
3. 684 ; 576.
4. A, \$125 ; B, \$225 ; C, \$150.
5. \$150 ; \$200 ; \$250.
6. 120 ; 140 ; 280.
7. 61 ; 122 ; 183 ; 244 ; 305.
8. 450 ; 600 ; 765. ($2\frac{1}{2} : 3\frac{1}{3} : 4\frac{1}{4} = 30 : 40 : 51$).
9. 153 bu.; 170 bu.; 187 bu.
10. 525 bu. wheat ; 350 bu. oats. ($\frac{1}{2} : \frac{1}{3} = 3 : 2$).
11. 36 boys. $\left(52\% \text{ No. pupils} = 39;\ \therefore 48\% \text{ No. pupils} = \dfrac{39 \times 48}{52}\right)$.
12. \$330. (If B received \$6, A would receive \$9, and C \$10; ∴ $\frac{10}{25}$ sum = \$132).

Exercise 145. Page 152

1. A, \$750; B, \$500.   2. A, \$60; B, \$144; C, \$96.
3. Son, \$3900; widow, \$4800; daughter, \$3300. (Sum willed = \$20000; assets = \$12000; each gets $\frac{12}{20}$ of the sum willed).
4. A, \$14.40; B, \$9.60.   5. A, \$6.30; B, \$5.25; C, \$4.20
6. A, \$10920; B, \$5460.   7. A, \$1080; B, \$1260; C, \$1440, D, \$1620. (Each gained 18% of his capital).
8. \$13275. ($\frac{90}{295}$ of total gain = \$4050).
9. \$1765. (C furnished $\frac{148}{383}$ of the capital).
10. B's profits = \$87; C's capital = \$320.
11. A, \$350; B, \$210; C, \$280. (C as manager gets $12\frac{1}{2}$% of \$840 = \$105; ∴ \$735 is divided in proportion to capital).
12. A, \$2333$\frac{1}{3}$; B, \$2500; C, \$2666$\frac{2}{3}$. (Total loss = \$7500).
13. 288 gal. of wine; 72 gal. of water.
14. A, \$2662; B, \$2420; C, \$2200; D, \$2000. (Suppose D gets \$1000).   15. A, 540 pages; B, 360 pages; C, 485 pages. {No. pages in A, B and C = $\frac{1}{2}$(900 + 845 + 1025) = 1385}.
16. 33$\frac{1}{3}$%. (A receives $\frac{105}{315}$ gain = $\frac{1}{3}$ gain).
17. A, \$15.60; B, \$14.40; C, \$8. (A, 30 sheep for 13 wk. = 390 sheep for 1 wk., etc.).

Exercise 146. Page 154

1. A, \$369.20; B, \$563.80; C, \$184.60. (A's share = twice C's share; B's share = 3 times C's + \$10; total sum = 6 times C's + \$10; ∴ 6 times C's share = \$1107.60).
2. 62$\frac{6}{7}$ yd.; 9$\frac{2}{7}$ yd.; 82$\frac{6}{7}$ yd. $\left(\text{Total No. yd.} = \frac{240.25}{1.55} = 155\right)$. No. yd. 1st bought = 3 times No. 2nd bought + 35 yd.; No. yd 3rd bought = 3 times No. 2nd bought + 55 yd.; ∴ 7 times No. 2nd bought + 90 yd. = 155 yd.).
3. \$12 for 1st; \$16 for 2nd; \$9 for 3rd. (Price of 1st = price of 3rd + \$3; price of 2nd = price of 3rd + \$7; total = 3 times price of 3rd + \$10 = \$37).
4. 63 yd.; 82 yd. (Cost of tweed = \$205; of silk = \$204.75).
5. Oats, 27c.; wheat, 69c. {If the wheat were the same cost a the oats, the cost would be (42 × 17)c. less, or \$14.97 − \$7.14 *or* \$7.83; ∴ cost of 29 bu. oats = \$7.83}

6. 14 ten-cent pieces: 12 five-cent pieces. (If No. of 10-cent pieces were same as No. of 5-cent pieces, Mary would have 20c. less than she has, or $1.80. Value of 1 coin of each = 15c.; ∴ No. of 5-cent pieces $= \frac{1.80}{.15}$).

7. 12 men. (If each received 92c., each boy would have an advance of 27c.; ∴ the 8 boys would get (27 × 8)c. more. Each man would get 18c. less; ∴ No. men $= \frac{27 \times 8}{18}$).

8. $112\frac{1}{2}$ ninths; $337\frac{1}{2}$ tenths; $562\frac{1}{2}$ twelfths. (If 1 ninth, 3 tenths and 5 twelfths, the sum $= \frac{149}{180}$, ∴ No. ninths $= 93\frac{1}{8} \div \frac{149}{180}$).

9. 10 horses; 30 cows. (Value of 1 horse and 3 cows = $200).

10. A, 26 bu.; B, $19\frac{1}{2}$ bu. ($\frac{7}{8}$ A's share = B's share + $\frac{1}{8}$ A's share; ∴ $\frac{3}{4}$ A's share = B's share; ∴ if A has 4 bu., B has 3 bu.).

Exercise 147. Page 155

1. (*a*) 1478; 295.6. (*b*) $28\frac{23}{40}$; $5\frac{143}{200}$. (*c*) 31.3; 6.26.
2. 34. 3. (*a*) 3010 ft. (*b*) $3656\frac{5}{8}$ ft. (*c*) $1408\frac{1}{4}$ ft.
4. 28.72. 5. 73.5 lb. 6. $14\frac{1}{2}$ lb. 7. 111 lb
8. $14.858\dot{3}$ ft. $= 14\frac{103}{120}$ ft. = 14 ft. $10\frac{3}{10}$ in. (Total $= 59.4\dot{3}$ ft.)
9. $1608\frac{3}{4}$ lb. (Total = 12870 lb 10. 8.8.
11. 15602 ft. 12. 65c. 13. $$2.06\frac{1}{4}$
14. 43°.9925. 15. 4 ft. 7 [illegible] 16. $44\frac{4}{5}$c
17. 225 bbl. @ $6.50 250 bbl. @ $6 [illegible] ($$6\frac{1}{5}$ [illegible] $$6\frac{3}{4} - $6\frac{1}{5} = $$ [illegible] 9 bbl [illegible] bbl @ $6.75)
18. $2 an oz. (8 oz. silver and 7 [illegible] gold [illegible] the value [illegible] $240 [illegible] of silver depreciat[illegible] $112 in second [illegible] because [illegible] depreciation $98. Ratio of [illegible] 112 : 98 = 8 : 7, first weight [illegible] gold in first; Ratio of weight of silver to gold [illegible] oz. silver and 7 oz. gold. Value of 7 oz. gold $16 [illegible] value of 8 oz. silver $128 [illegible] $2 - $16.
19. $12 an acre. (Cost of 320 ac. $5200 [illegible] 240 ac. = $4240; of 80 ac. [illegible]) [illegible] $42 [illegible]

Exercise 148. Page 157

11. (*a*) 1 : 3. (*b*) 1 : 5. (*c*) 1 : 6. (*d*) 16 : 3. (*e*) 3 : 1. (*f*) 10 : 7;
12. (*a*) 9. (*b*) 84. (*c*) 8. (*d*) $1\frac{1}{2}$. (*e*) 10. (*f*) .2.
13. 8 : 4 = 12 : 6. 14. 4 : 12 = 5 : 15. 15. 20 : 5 = 24 : 6
16. (*a*) 36 : 12 = 6 : 2. (*b*) 56 : 7 = 24 : 3. (*c*) 25 : 5 = 55 : 11.
(*d*) 5 : 8 = 25 : 40. (*e*) $1\frac{3}{4}$: $\frac{1}{6}$ = 21 : 2. (*f*) 54 : 9 = 72 : 12.
20. (*a*) 12 : 6 = 4 : 2. (*b*) 35 : 6 = 4 : $\frac{24}{35}$. (*c*) 5 : 8 = 10 : 16.

Exercise 149. Page 159

1. 375 lb.	2. 44.625 ft.	3. 273 bu.	4. $16.
5. $720.	6. $70.	7. $3\frac{1}{3}$ da.	8. $$1\frac{3}{32}$.
9. 150 men.	10. $12187.50.	11. 288 mi.	12. 34 lb.
13. $25.	14. $$20.41\frac{2}{3}$		

Exercise 150. Page 160

1. $143.65; 25%. 2. $3000. 3. $8\frac{8}{9}$ da.
4. 20; 40; 60. 5. He still gains 10% of cost.
6. $3\frac{11}{18}$ da. 7. $37.30. 8. 2880 ft. board measure.
9. 94885. (Dividend = $\frac{1}{4}$ divisor × 948 + 85).
10. $$5484.37\frac{1}{2}$. 11. $76.85.
12. 900 men; 1800 men; 2100 men. (6 times No. men in 1st regiment = 4800 + 600). 13. $\frac{5}{7}$. 14. $278.50.
15. $1.25. ($3\frac{1}{2}$% of $\frac{5}{8}$ value = $175). 16. $5\frac{1}{5}$%.
17. 48 yr.; 16 yr.
18. $6.37. (Cost of 3 bu. @ $1.75 = $5.25; of 2 pk. = $\frac{1}{2}$ of $1.75 = $.875; of 1 gal. $\frac{1}{4}$ of $.875 = $.21875; of 1 pt. = $\frac{1}{8}$ of $.21875 = $.02734375. Total cost = $6.37109375).
19. $9\frac{4}{9}$%. (Net annual receipts = $32.50 × 12 − $50 = $340).
20. 999.99475. (10000 − .0525) × .10.
21. $$29\frac{1}{6}$; $$33\frac{5}{6}$. 22. 87357. 23. $5\frac{7}{25} = 5.28$.
24. $744. ($109 = amount of $100 in $1\frac{1}{2}$ yr. @ 6%).
25. $273.75. 26. $4.9\dot{1}5042\dot{3}$. 27. $1.9\dot{4}9\dot{0}$.

Exercise 152. Page 164

1. 529. 2. $650\frac{1}{4}$. 3. $300\frac{4}{9}$. 4. 9409.
5. $6\frac{1}{4}$. 6. $297\frac{9}{16}$. 7. 72.25. 8. 62.41.
9. 134689. 10. $1320\frac{1}{9}$. 11. $\frac{25}{64}$. 12. $55\frac{1}{144}$.

Exercise 153. Page 164

1. 6859. 2. 421875. 3. 658503. 4. 857357.
5. 91.125. 6. $12\frac{19}{27}$. 7. $32\frac{96}{125}$. 8. 405.224
9. 8869743. 10. $47634\frac{49}{64}$. 11. 14706.125. 12. $376\frac{521}{729}$.

Exercise 154. Page 164

1. 8836. 2. 15625. 3. 4096. 4. 7225.
5. .0049. 6. .003375. 7. 15.625. 8. .01185921.
9. $3\frac{1}{16}$. 10. $\frac{16}{81}$. 11. $12\frac{19}{27}$ 12. 1[illegible]

Exercise 155. Page 164

1. 72. 2. 500. 3. 64. 4. 4913.
5. 8. 6. 405. 7. 2187. 8. 1250000.
9. $\frac{3}{2}$. 10. $\frac{1}{768}$. 11. 1.30321.
12. $10^6 = 1000000$.

Exercise 156. Page 164

1. 169; 196; 225; 256; 289; 324; 361; 400.
2. 1331; 2197; 3375; 4913; 6859. 3. Eighth power.
4. Fifth power. 5. $(5007)^2 = 25070049$.
6. $(202)^3 = 8242408$. 7. $(.5)^4 = .0625$.
8. 2025 sq. rd. (Side of sq. field = 45 rd.).
9. Third power. 10. 4; 729; 262144.
11. Sixth power. 12. $\frac{1}{2}$.

Exercise 157. Page 165

1. 25; 81; 144. 2. 6; 7; 11; 9. 3. 3; 5; 8; 12.
4. 4 × 4; 5 × 5; 9 × 9; 7 × 7; 10 × 10. 5. 3; 6; 8; 12.
6. Not more than two. 7. Either three or four.
8. Either five or six. 9. Three.

10. 2; 2; 2; 3; 2; 3. 11. 2; 3; 2; 1; 4.

12. Either 5 or 6.

13. The No. of figures in the sq. root is the same as the No. of *periods* of the two figures each in the No.

Exercise 158. Page 166

2. 1849. 3. 3136.

4. $(\text{Tens})^2 + 2 \times \text{tens} \times \text{units} + (\text{units})^2$.

5. $(40)^2 + 2 \times 40 \times 7 + 7^2$. 6. $(70)^2 + 2 \times 70 \times 8 + 8^2$.

Exercise 159. Page 167

1. 17. 2. 19. 3. 24. 4. 25. 5. 36.

6. 75. 7. 95. 8. 49. 9. 64. 10. 37.

11. 47. 12. 56.

Exercise 160. Page 168

1. 625. 2. 512. 3. 343. 4. 707.

5. 906. 6. 857. 7. 2401. 8. 6325.

9. 5008. 10. 4385. 11. 6671. 12. 7854.

Exercise 161. Page 168

1. 40 rd. 2. $2560. (Side of field $= \sqrt{}$160 rd.).

3. 30 ft. square. (Area = 100 sq. yd.; side $= \sqrt{100}$ yd. = 10 yd.).

4. 125.

5. 66 yd. long, 22 yd. wide. (Three times square of width = 1452 sq. yd.; square of width = 484 sq. yd.; width = 22 yd.).

6. 63 yd. long, 42 yd. wide.

7. 85 yd.; 85c. per yd. (No. yd. $= \sqrt{7225}$).

8. 246 ft. 11.39 ... in. (Area of square = 8781696 sq. in.).

9. 250 ft. long, 125 ft. wide. 10. 57. $(\sqrt{361 \times 9})$.

11. 20 yd. long, 4 yd. wide. (No. sq. yd. $= \frac{19 \times 20 \times 12d.}{57d.}$; five times square of width = 80 sq. yd.).

12. $1496. (Side of field = 40 rd.).

Exercise 162. Page 169

1. 300 ; 160. (Greater No. = $\frac{15}{8}$ smaller No.; ∴ $\frac{7}{8}$ smaller No. = 140).

2. 50 rd. $\left(88 \text{ ft.} = 5\frac{1}{3} \text{ rd.} \quad \text{Length} = \frac{1\frac{2}{3} \times 160}{5\frac{1}{3}} \text{rd.}\right)$.

3. 6 min. (Area of field = 3600 sq. rd.; side = 60 rd.; perimeter = 240 rd. = $\frac{3}{4}$ mi. To ride $7\frac{1}{2}$ mi. takes 60 min.; to ride $\frac{3}{4}$ mi., etc.).

4. $6. ($2\frac{1}{2}$% cost of flour = $375). 5. 55.07 ; 792.

6. $2\frac{1}{2}$ da. (12 boys do as much work per day as 6 men ; 8 boys do as much per day as 4 men: ∴ 8 men and 8 boys take same time as 12 men).

7. 6 hr. 40 min. $8\frac{14}{15}$ sec. (A difference of 15° in longitude gives a difference of 1 hr. in time).

8. $15600. $\left(\frac{12\frac{1}{2}}{1000} \text{ of } \frac{3}{4} \text{ value} = \$146.25\right)$.

9. 20 min. 36 sec. past 4 p.m (Diff. in longitude = 132° 39′).

10. 111 yd. long, 37 yd. wide. 11. $1500 ; $2100 ; $1800.

12. 105 bbl.; 175 bbl.

13. 12520 yd. (S.p. = $3300; 1st com. = $88; sum paid for cloth = $3300 − $88 − $82).

14. $17.29. {Amount of $1 computed yearly = $$(1.08)^2$ = $1.1664: computed half-yearly = $$(1.04)^4$ = $1.16985856; difference for $1 = $.00345856}.

15. $1675\frac{1}{3}$ bu. wheat.

16. $\frac{83}{4}$; $\frac{1}{16}$; $\frac{15}{7}$. 17. $12\frac{1}{24}$ mi. (Rate *up* stream = $2\frac{7}{8}$ mi. an hr).

18. 6 hr. {Part filled in 1 hr. = $(\frac{1}{3} + \frac{1}{4} - \frac{1}{2})$ cistern}.

19. $30 (S p. = $\frac{9}{10}$ of $40 = $36 = $\frac{6}{5}$ cost). 20. $6\frac{2}{3}$%.

21. $194\frac{2}{3}$. ($12\frac{1}{2}$% No. = $24\frac{1}{3}$).

22. (*a*) 108 yd. (*b*) $69\frac{9}{14}$. (*c*) 280 bu.

23. $510.20. (98% face of note = $500).

Exercise 164. Page 174

1. $2\frac{3}{16}$ sq. yd. ($1\frac{3}{4}$ yd by $1\frac{1}{4}$ yd.). 2. $114\frac{6}{11}$ sq. rd.

3. $91.94. $\left(\$45 \times \frac{87 \times 62}{16\frac{1}{2} \times 160}\right)$.

4. $41.10. (See Exercise 52, page 147). 5. $316.80.

6. 132. (Perimeter = 96 rd.). 7. $5\frac{1}{3}$ ft. (See Ex. 53, page 147).
8. $328\frac{7}{36}$ sq. ft. = 328 sq. ft. 28 sq. in.
9. 24 rd. long; 18 rd. wide. (Area = 432 sq. rd. If length were the same as the width, the area would be $\frac{3}{4}$ as great, or 324 sq. rd.; ∴ width = $\sqrt{324}$ rd. = 18 rd.).
10. $\$35 = \$25 \times \frac{6 \times 7}{5 \times 6}$.
11. 2560 ac. (Area = 2 mi. sq. = 4 sq. mi = 640 × 4 ac.).

Exercise 165. Page 17

1. $12\frac{1}{3}$ yd. by $6\frac{1}{6}$ yd. 2. \$13.20. (Per. = 90 ft.; No. strips $= \frac{90 \times 12}{18} = 60$; length of strip = 4 yd.; No. rolls $= \frac{60 \times 4}{8} = 30$. For ceiling, No. strips = 14; length of strip = 24 ft. = 8 yd.; No. rolls = 14; ∴ total No. rolls = 44).
3. 200 rd. (If length were only as great as the width the area would be $\frac{5}{8}$ as great, or 10 ac., or 1600 sq. rd.; ∴ width = $\sqrt{1600}$ rd. = 40 rd.). 4. 10 yr. (No. drains = 15; length of each = 1320 ft.; cost = \$396. No. ac. = 30; value of extra yearly yield = \$39.60). 5. \$23.25. (No. ac. = 108; total cost = \$3688.20; net cost = \$2511).
6. 12130560 ac. (4′ 6″ represents 162 mi.; 3′ 3″ represents 117 mi.).
7. 15360 ac. 8. \$320.64. (Length of walk = 2672 ft.).
9. \$132. (Perimeter = 80 rd. = 1320 ft.). 10. $\$96\frac{5}{8}$.
11. \$281.65 (nearly). (Per. of rect. field = 340 rd.; area = 98 × 72 sq. rd.; side of sq. field = $\sqrt{98 \times 72}$ rd. = $\sqrt{49 \times 2 \times 2 \times 36}$ rd. = 7 × 2 × 6 rd. = 84 rd.; per. = 336 rd.; cost = $\$285 \times \frac{336}{340}$).

The Parallelogram. Page 176.

2. (*a*) $8\frac{1}{3}$ sq. ft.; (*b*) 390.975 sq. ch. = 39.0975 ac.; (*c*) $6\frac{206}{605}$ ac. = 6 ac. 54 sq. rd. 14 sq. yd. 4 sq. ft. 72 sq. in.
3. 24 ft. 6 in.; 2.5 ch.; $318\frac{3}{8}$ ac. (1 ch. = 4 rd.).
Page 177—1. $114\frac{8}{11}$ sq. rd. (Ex. 2, page 174).
2. (Ex. 3, page 174). \$91.94. 3. 6. 4. 192 sq. ft.

Exercise 166. Page 177

7. 27 sq. in. 8. 120 sq. in. 9. $217\frac{1}{3}$ sq. in.

Exercise 167. Page 179

1. $42\frac{1}{4}$ sq. yd. 2. $56\frac{5}{8}$ sq. yd.
3. 44 sq. rd. (Base = 12 rd.; perp. = $7\frac{1}{3}$ rd.).
4. 15 ft. 5. $21\frac{1}{4}$ ch. = 21 ch. 25 links. (1 ch. = 100 links).
6. 220 yd. 7. 36.4 ch. = 36 ch. 40 l. = 3640 l.

Exercise 168. Page 180

1. 37 ft. 2. $64\frac{5}{6}$ ft. = 64 ft. 10 in. 3. 650 ft. 4. 77 in.
5. 44 ft. 6. 70 ch. 7. 4 ft. 8. 390 yd.
9. 56 ft. (Hyp. = 65 ft.; base = 33 ft.). 10. 4 ch. 80 l.
11. .61105 sq. in. (Δ A C D = $\frac{1}{2}\times 1.26\times .55$ sq. in.; Δ D C F = $\frac{1}{2}\times .95\times .4$ sq. in.; Δ C F H = $\frac{1}{2}\times .71\times .21$ sq. in.).

Exercise 169. Page 180

1. 40 ft. = $\sqrt{(32)^2+(24)^2}$ ft. 2. 20 ft.
3. 13 ft. = $\sqrt{(85)^2-(84)^2}$ ft.
4. 96 ft. (Stump = $\sqrt{(51)^2-(24)^2}$ ft. = 45 ft.). 5. 193 ft.
6. 114 ft. = $\sqrt{(190)^2-(152)^2}$ ft.
7. 4 ft. (Top of ladder still reaches $\sqrt{(34)^2-(16)^2}$ ft. from the ground). 8. 34 ft. = $\sqrt{(16)^2+(30)^2}$ ft.
9. $487.20. (Perp. = 638 ft.; per. = 2320 ft.).
10. 85 ft. = $\sqrt{(60)^2+(32)^2+(51)^2}$ ft.
11. 6 ft. 3 in. = $\sqrt{5^2+3^2+(2\frac{1}{4})^2}$ ft.

Exercise 171. Page 182

1. 110 ft. 2. 286 ch. 3. 44 ft. 4. 33 yd. = 6 rd.
5. 11 ch. 6. 24 ch. 64 l. 7. $62\frac{6}{7}$ ft. 8. 14 ch. 96 l.
9. 17 yd. 4 in. 10. 14 in. 11. $38\frac{1}{2}$ in. 12. 35 in.
13. [illegible] 14. 1120 ft. 15. 4.9 mi. 16. $20\frac{4}{11}$ rd.
17. 1 ft. $71\frac{1}{2}$ 18. 70 ft. 19. $12\frac{4}{7}$ ft.

20. 14 in. (Side of square = 22 in.; circumference of circle = 88 in.; radius = 14 in.). 21. $158.40. (Circum. = 44 yd.).

22. 8. $\left(\frac{60 \times 1760 \times 3}{11 \times 60 \times 60}\right.$ times per sec. Train goes 11 ft. per rev.$\left.\right)$.

Exercise 172. Page 183

5. 154 sq. in. 6. 154 sq. in. 7. 616 sq. in.

Exercise 173. Page 183

1. 5544 sq. in. = $38\frac{1}{2}$ sq. ft. 2. 7546 sq. yd.
3. $24062\frac{1}{2}$ sq. yd. 4. $13898\frac{1}{2}$ sq. ft.
5. 18.32985 sq. ch. 6. 1386 sq. yd.
7. $471\frac{5}{8}$ sq. in. (R. = $12\frac{1}{4}$ in.). 8. 647.185 sq. ch. (R. = 14.35 ch.).
9. 3850 sq. yd. (R. = 35 yd.). 10. $\frac{43}{56}$ sq. rd. (R. = $\frac{3}{4}$ rd.).
11. 17796.625 sq. l. (R. = $75\frac{1}{4}$ l.).
12. $\frac{7}{88}$ sq. mi. (R. = $\frac{7}{44}$ mi.; $\frac{7}{88}$ sq. mi. = $50\frac{10}{11}$ ac.).
13. $38\frac{1}{2}$ sq. ft. 14. $180\frac{53}{72}$ sq. ft. = 180 sq. ft. 106 sq. in.
15. 154 sq. ch. 16. $9\frac{5}{8}$ sq. rd. 17. $346\frac{1}{2}$ sq. yd.
18. 3850 sq. ch. = 385 ac.

Exercise 174. Page 184

1. $176\frac{1}{14}$ sq. yd. 2. $\frac{7}{88}$ sq. mi. (See Ex. 12, page 184).
3. $150\frac{4}{7}$ ft. 4. $412\frac{1}{2}$ ft.
5. 27 in. $\{\pi R^2 = \pi(8^2+9^2+12^2)$; $\therefore R = \sqrt{8^2+9^2+12^2}\}$.
6. $1013.76. (R. of lawn = 110 yd.; R. outer circum. of path = 114 yd.; area of path = $\pi\{(114)^2-(110)^2\}$ sq. yd. = $\frac{22}{7} \times 896$ sq. yd.). 7. 77 sq. in. ($\frac{1}{2}$ large circle is covered).
8. Circum. = 396 in. When angle = 50°, arc = $396 \times \frac{5}{36}$ in. = 55 in.; angle = 65°, arc = $396 \times \frac{6.5}{36}$ in. = $71\frac{1}{2}$ in.; angle = 80°, arc = 396 $\times \frac{8}{36}$ in. = 88 in.; angle = 90°, arc = $396 \times \frac{9}{36}$ = 99 in.
9. 62.048 ... yd. (Area = $2\frac{1}{2}$ ac. = $2\frac{1}{2} \times 160 \times 30\frac{1}{4}$ sq. yd. = $\frac{22}{7}r^2$; $\therefore r^2 = 3850$ sq. yd.).
10. $556\frac{2}{7}$ sq. in. [R. of larger circle = 31 in.; of smaller = 28 in.; area = $\frac{22}{7}\{(31)^2-(28)^2\}$ sq. in.].
11. $21\frac{21}{32}$ sq. ft. (Dia. of circle = $5\frac{1}{4}$ ft.).
12. $75\frac{3}{7}$ sq. in. (Dia. of one cent coin = 1 in.; area of circle = 154 sq. in.; area of 100 cents = $78\frac{4}{7}$ sq. in.).

Exercise 175. Page 185

1. (*a*) $121\frac{1}{2}$ sq. in.; (*b*) $91\frac{1}{4}$ cu. in. 2. (*a*) $86\frac{2}{3}$ sq. ft.; (*b*) 50 cu. ft.
3. (*a*) 138 sq. in.; (*b*) $87\frac{1}{4}$ cu. in. 4. 216. 5. 4320 sq. in.
6. $14\frac{2}{7}$ ft. 7. 2197 cu. in. (Area of cube = 1014 sq. in.; of 1 face = 169 sq. in.; edge = 13 in.). 8. $\frac{5}{256}$ in.
9. $16\frac{2}{3}$ ft. $\left(\text{Length cut off} = \frac{2\frac{1}{4} \times 1728}{18 \times 15}\text{ in.}\right)$.
10. $1\frac{23}{25}$ ft. ($6\frac{1}{4}$ gal. = 1 cu. ft.; ∴ 40 cu. ft. water). 11. $2\frac{2}{3}$ ft.

Exercise 176. Page 186

1. 36 ft. (Area of floor = 864 sq. ft. If length were only as great as the width the area would be $\frac{2}{3}$ of 864 sq. ft. = 576 sq. ft.; ∴ width = $\sqrt{576}$ ft. = 24 ft.). 2. $6\frac{1}{4}$ ft.
3. $1107\frac{23}{36}$ gal. (Internal dimensions are 9′ 8″, 3′ 8″, 5′).
4. 10 in. 2(15 × 14 + 15 depth + 14 depth) = 1000 sq. in
5. 8640. 6. $4\frac{4}{5}$ ft. $\left(\text{Length cut off} = \frac{11 \times 1728}{22 \times 15}\text{ in.}\right)$.
7. $3\frac{1}{5}$ ft. $\left(1000\text{ gal.} = \frac{1000}{6\frac{1}{4}}\text{ cu. ft.} = 160\text{ cu. ft.}\right)$.
8. 18 ft. $\left(\text{Depth} = \frac{584.50 \times 27}{.25 \times 125\frac{1}{4} \times 28}\text{ ft.}\right)$.
9. $\frac{1}{16}$. $\left(\text{Thickness} = \frac{1728}{48 \times 12 \times 4 \times 12}\text{ in.}\right)$.
10. $5\frac{4}{7}$ ft. 11. 4515 lb. 12. 500 loads. (Weight of $1\frac{1}{10}$ cu. ft. ice = 1000 oz. = $62\frac{1}{2}$ lb.; weight of 1 cu. ft. ice = $\frac{125}{2} \times \frac{10}{11}$ lb.).

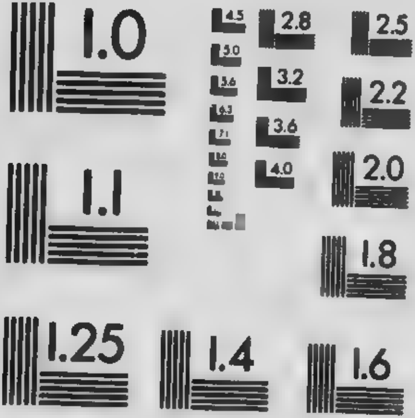
Exercises 177 — Oral

Exercise 178. Page 188

1. 8052 sq. in. (Curved surface = 5280 sq. in.; ends = 2772 sq. in.).
2. $129\frac{1}{4}$ “ “ “ = 110 “ ; “ = $19\frac{1}{4}$ “
3. 528 sq. ft. “ “ = 220 sq. ft.; “ = 308 sq. ft.
4. $833\frac{1}{4}$ “ “ “ = 660 “ ; “ = $173\frac{1}{4}$ “
5. 957 “ “ “ = 264 “ ; “ = 693 “
6. $207\frac{17}{56}$ “ “ “ = $102\frac{2}{7}$ “ ; “ = $104\frac{29}{56}$ “

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



APPLIED IMAGE Inc
1653 East Main Street
Rochester, New York 14609 USA
(716) 482 - 0300 - Phone
(716) 288 - 5989 - Fax

7. 1848 cu. in. (Measure of the vol. = measure of area of end multiplied by measure of height).

8. 55440 cu. in. = $32\frac{1}{12}$ cu. ft. 9. 20790 cu. in.

10. 770 cu. ft. 11. 2079 cu. ft. 12. $962\frac{1}{2}$ cu. ft.

13. 7 in. $\left(\text{Since vol.} = \pi r^2 h\ ;\ \therefore\ r = \sqrt{\dfrac{\text{Vol.}}{\pi h}}\right)$

14. 11 in. 15. 7 ft. 16. $3\frac{1}{2}$ ft. 17. $10\frac{1}{2}$ ft.

18. 14 ft.

Exercise 179. Page 189

1. 14 in. (Curved surface = $\pi d h$). 2. 66000 sq. ft.

3. 6600. 4. $24.75. 5. 1 ft. 4 in. (No. sq. ft. = 88 ; circum. = $\frac{88}{21}$ ft.; dia = $\frac{88}{21} \times \frac{7}{22}$ ft.).

6. $19\frac{1}{4}$ ft. [Vol. copper = $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$ cu. in. = 462 cu. in. Length = $\{462 \div (8 \times \frac{1}{4})\}$ in. = 231 in.]. 7. 6912.

8. $220. 9. $3019\frac{1}{2}$ lb. 10. $848\frac{4}{7}$ cu. ft.

Exercise 180. Page 189

1. $3\frac{3}{4}$ ft. board measure. (Average width = 9 in.).

2. 7 sq. in. = $\left(\dfrac{3+4}{2}\right) \times 2$ sq. in. 3. $35\frac{1}{16}$ tons.

4. $282\frac{6}{7}$ cu. ft. 5. $57042\frac{6}{7}$ cu. ft. (Dia. = 5 ch. = 20 rd. = 330 ft).

6. $12305\frac{7}{10}$ t. 7. $303. 8. $186\frac{3558}{3697}$ bu. 9. 12031250 gal.

10. 80 ft. (Area of triangle = $\frac{1}{2}$ area of rectangle of same base and height).

11. 916.5 ... sq. rd. (Altitude = $\sqrt{(50)^2 - (20)^2}$ rd. = $\sqrt{2100}$ rd. = 45.825 ... rd.).

12. 16.85 ... ft. = $\sqrt{(72)^2 - (70)^2}$ ft.

13. 1452 sq. ft. (When the sides are 3 ft. and 4 ft., the diag. is 5 ft.; ∴ sides are 33 ft. and 44 ft.).

14. 7.07 ... in. (Diag. of sq. = 10 in.; $2(\text{side})^2 = (10)^2$; ∴ side = $\sqrt{50}$ in.).

15. 43.08 ... ft. = $\sqrt{(32)^2 + (24)^2 + (16)^2}$ ft.

16. $18. (Per. of sq. field = 320 rd.; area = 40 ac.; length of rect. field = 100 rd.; width = 64 rd.; per. = 328 rd.).

17. 3456. (Each shingle covers 72 sq. in.).
18. 490 ft. board measure. 19. $894\frac{419}{521}$ gal.
20. $87.50. Area $=\left(\frac{40+100}{2}\right)\times 75$ sq. ft. 21. $15\frac{6}{13}$ sq. ft.
22. 75579.82 ... gal. 23. $37\frac{5}{7}$ in.; $\frac{11}{105}$ in. 24. 18 ft.
25. Area of sq. $=110\frac{1}{4}$ sq. ft.; of circle $=140\frac{7}{22}$ sq. ft.; ∴ circle by $30\frac{3}{44}$ sq. ft. 26. $519.75.
27. 229.25 ... mi. $=12\sqrt{(13)^2+(14)^2}$ mi.
28. (*a*) $48. (*b*) $57.60.
29. 7 ft. (Value of scantling = $6 ; ∴ No. ft. plank, board measure = 1680).
30. $56. (Additional cost is the cost of plank for 500 ft. in length and 2 ft. wide). 31. $36\frac{3}{4}$ sq. yd.
32. (*a*) 1 hr. 20 min. (*b*) $2\frac{239}{360}$ ac. = 2 ac. 106 sq. rd. 6 sq. yd. 7 sq. ft. 72 sq. in. (*c*) $462\frac{2}{5}$ ac. (24 rd. have been cut away around the field).
33. $24. 34. $33. 35. $62.40. 36. $255.64.
37. $71.22. (No. posts = 40, cost = $10 ; No. pickets = 440, cost = $19.36 ; scantling = 440 ft., cost = $11.88 ; base board = 165 ft., cost = 4.45\frac{1}{2}$; rain board = 165 linear ft., cost = $4.95 ; lumber = 660 ft., cost = $17.82; nails, $2.75; total = 71.21\frac{1}{2}$).

Exercise 181. Page 193

1. 262144 cu. in. (Area of 1 face $=\frac{1}{6}$ of 24576 sq. in. = 4096 sq. in. ; ∴ edge of cube = 64 in.).
2. Watch must be "put on" 6 hr. 59 min. $33\frac{7}{15}$ sec. for exact solar time, but 7 hr. for standard time. 3. $25.
4. $99\frac{5}{6}$ yd. (No. strips $=\frac{64\times 12}{32}=24$; No. of patterns per strip = 10, and 6 in. waste per strip ; 1st strip is 12 ft. long ; the other 23 are each cut $12\frac{1}{2}$ ft. long ; ∴ total length $=299\frac{1}{2}$ ft.).
5. $865.49. (Due Sept. 19 ; term of dis. = 73 da.).
6. $18600. 7. $8\frac{1}{3}$%. 8. $105. 9. 97.24 ... ; 5.72.
10. $9216=2^{10}\times 3^2$; sq. rt. $=2^5\times 3=96$.
11. $\$10653.66=\$10920\times\frac{100}{102\frac{1}{2}}$. 12. $45\frac{615}{907}$c.

13. $2.50. $\left(\text{No. strips} = \frac{54 \times 12}{18} = 36\right.$; length of 1st strip = 8 ft., and of each of the other 35 strips cut $8\frac{1}{3}$ ft.; No. yd. = $99\frac{8}{9}$).
14. $108.06. $\left(\text{No. strips} = \frac{14\frac{1}{2}}{1\frac{1}{2}} = 10\right.$; 1st strip is cut 18 ft. long, and each of the other 9 is cut 18 ft. 7 in. Total length = $185\frac{1}{4}$ ft. = $61\frac{3}{4}$ yd.).
15. (*a*) 5 strips. (Carpet to run lengthwise); (*b*) 9 in.; (*c*) $30.29, i.e., $20\frac{8}{9}$ yd. @ $1.45).
16. $6 loss. (Cost of 1st = $168; of 2nd = $216).
17. $14 gain; 35% gain. S.p. = $280; gain = $60. Cost = $123; gain = $41. Cost = $42; gain % = $33\frac{1}{3}$.
18. $\$812.18 = \$800 \times \frac{100}{98\frac{1}{2}}$. 19. $50\frac{10}{11}$ ac. (R. = $320 \times \frac{7}{44}$ rd.).
20. $4\frac{1}{2}$%. 21. $44000. 22. $18000. (A's capital is $\frac{12}{5}$ of B's).
23. $384.75. 24. $6\frac{6}{19}$ ft. = 6 ft. $3\frac{15}{19}$ in.

Exercise 182. Page 195

1. (*a*) 31c.; (*b*) 55c.; (*c*) 69c. 2. $2.25. 3. $26.60.
4. 24 da. 5. 45 mi. an hr. 6. 26 ft. 8 in. (1 cu. ft. = $6\frac{1}{4}$ gal. = $\frac{25}{32}$ bu., *or* 1 bu. = $\frac{32}{25}$ cu. ft.). 7. 1[illegible] lb.
8. $126.50. 9. $580. 10. $14.50, $20\frac{5}{6}$%; $6.90, 90c.; $14, $32\frac{1}{7}$%; $\$17.28\frac{104}{107}$, $\$1.21\frac{3}{107}$; $310, $322.40.
11. 12 ton. 12. (*a*) 75c.; (*b*) 75c.; (*c*) $1.20.
13. $20\frac{10}{27}$%. (Long ton = 2240 lb.).
14. (*a*) 7776[illegible] (*b*) $2656.80; (*c*) 26.568%. 15. 250 men.
16. (*a*) $1\frac{7}{20}$ *b*) 4050 lb.; (*c*) $5872.50.
17. $6\frac{1}{4}$ mi. {To *go* and *return* 1 mi. requires $(\frac{1}{25} + \frac{2}{15})$ hr.}.
18. Canada, 1198.4 bu.; Great Britain, 1780.8 bu.; France, 1456 bu.; Germany, 1086.4 bu.; United States, 750 bu.; Argentina, 1041.6 bu.
19. (*a*) 400 ac. (*b*) 32 da. (*c*) $1470. (*d*) $4380.
20. $3\frac{1}{3}$ t. 21. (*a*) $10\frac{1}{2}$ t. (*b*) $$76.12\frac{1}{2}$.
22. 10 yr. (15 drains each 80 rd., cost $396).
23. $23.25. 24. 12130560 ac. 25. $66.
26. $3.60. (Butter fat increased 20 lb. daily).

Exercise 183. Page 198

1. 61.67 ... %.
2. 17.94 ... bu. per ac. in 1891 ; 9.33 ... bu. per ac. in 1901.
3. (*a*) 20.49 (nearly) cents per lb. for butter. (*b*) 7.04 ... c. is price of cream per lb. paid to patrons.
4. (*a*) 1841-1851, 143 mi.; 1851-1861, 1987 mi.; 1861-1871, 549 mi.; 1871-1881, 4636 mi.; 1881-1891, 6507 mi.; 1891-1901, 4302 mi. (*b*) $893\frac{3}{4}$ % ; $1249\frac{109}{159}$ % ; 25.58 ... % ; 172.02 ... % ; 88.76 ... % ; 31.08 ... %. (*c*) 1851 to 1861.
5. (*a*) 34.81 ... %. (*b*) 43.18 ... %. 6. $3903.82 ... .
7. (*a*) $7463.04 ... . (*b*) Electric 1.91 ... times as expensive as steam, *or* $3559.22 per mile more than steam.
8. $7.43 ... . 9. 1197.1 ; $18.52 ... .
10. (*a*) $333794. (*b*) 8.606 ... %.
11. (*a*) In 1904, loss, 7.05 ... % ; in 1905, gain, 57.97 ... % (*b*) $9219945. (*c*) 1.25 ... %.
12. (*a*) Percentage in increase of population : 1881-1891, 131.23 ...; 1891-1901, 67.34 ... . For occupiers of land the percentages are : 148.66 ... ; 43.96 ... . Therefore the occupiers of land increased more rapidly during the first decade, and the population during the second decade. (*b*) 13.76 ... % ; 14.80 ... % ; 12.73 ... %.
13. .1199 ... engines, .05302 ... passenger cars, 2.3616 ... freight cars.
14. 12c. very nearly. (For 5 yr., total quantity = 138191108 lb., value = $16558431).
15. (*a*) $398.87 in 1891 ; $367.87 ... in 1901. (*b*) $31 decrease.
16. 1.4351 ... metres.
17. (*a*) $68235090. (*b*) Timber and lumber, $80129280. (*c*) $343053480.
18. (*a*) 15.75 (nearly) bu.; 9.63 (nearly) bu. (*b*) 365.92 ... %. (*c*) 184.75 ... % in total increase.
19. (*a*) $42\frac{5}{6}° = 42.83°$. (*b*) 58.7° ... . 20. 56.42 ... %.
21. $372.78 ... in 1901 ; $331.22 ... in 1891 ; $41.56 ... increase.
22. 84%.
23. $15.74 ... per oz.

Exercise 184. Page 202

2. 6558 = $\overline{\text{V}}$MDLVIII; 244 = CCXLIV; 2683 = MMDCLXXXIII.
The dash over the symbol increases its value 1000 times.
4. 25%.
5. $7056 = 2^4 \times 3^2 \times 7^2$; ∴ sq. rt. $= 2^2 \times 3 \times 7 = 84$.
6. 720 revolutions.
7. 7 ft.
8. $\frac{117}{20000}$; $\frac{1}{120}$.
9. 16 : 35. ($9\frac{3}{7} : 20\frac{5}{8} = \frac{66}{7} : \frac{165}{8} = \frac{2}{7} : \frac{5}{8} = \frac{16}{56} : \frac{35}{56} = 16 : 35$).
10. 127.27 ... ft. $= \sqrt{(90)^2 + (90)^2}$ ft.

Exercise 185. Page 202

1. 59520.4. $(.0018 + .000106 + .84 + .0509) \div .000015$.
2. 80.2285 ... lb. (1 bu. $= 277.27 \times 8$ cu. in. $= 2218.16$ cu. in.).
3. 1st No. + 2nd No. = 100; 2 times 1st No. − 2nd No. = 10; ∴ 3 times 1st No. = 110; 1st No. = $36\frac{2}{3}$; 2nd No. = $63\frac{1}{3}$.
4. $\$128 = \$176 \times \frac{10}{11} \times \frac{4}{5}$.
5. $35\frac{7}{9}$%.
6. $\$4901.96 = \$5000 \times \frac{100}{102}$.
7. \$9500.
8. 24 ; 32 ; 48. ($2\frac{1}{2} : 3\frac{1}{3} : 5 = 15 : 20 : 30 = 3 : 4 : 6$).
9. $2\frac{22}{25}$ ft. = 2 ft. $10\frac{14}{25}$ in. $= \dfrac{4275}{62\frac{1}{2} \times 7\frac{1}{2} \times 3\frac{1}{6}}$ ft.
10. $43\frac{1}{5}$ min. $= \dfrac{10 \times 1728}{25 \times 16}$ min.
11. 142 yd. $= \dfrac{35\frac{1}{2}}{36}$ of 144 yd.
12. 35.775 ft.
13. \$62.40.
14. $13\frac{3}{4}$ in.

Exercise 186. Page 204

1. 7.864 ... oz.
2. $172\frac{1}{8}$ sq. ft. $= 19\frac{1}{8}$ sq. yd.
3. $16\sqrt{3}$ sq. in. = 27.7 ... sq. in. (Altitude $= \sqrt{8^2 - 4^2}$ in. = 6.929 ... in.).
4. $36\sqrt{3}$ sq. in. = 62.35 ... sq. in. (Altitude $= \sqrt{12^2 - 6^2}$ in. = 10.392 ... in.).
5. 5280 sq. ft.
6. 80. (*One* of each would cost \$45.50).
7. 6912.
8. 896 : 165.
9. 10 hr. 25 min.
10. $13794\frac{1}{2}$ sq. ft.
11. 144 sq. in. (Area of trapezoid $= \frac{3}{4}$ area of the original triangle).
12. 2940 lb. $= 70 \times 35 \times 16 \times 62\frac{1}{2} \times .0012$ lb.
13. 3600 lb. $= 240 \times 60 \times \frac{25}{100}$ lb.

Exercise 187. Page 207

1. 10 ; 100 ; 1000 ; 10 ; 100.

2. $\frac{1}{10}$ *or* .1 ; $\frac{1}{100}$ *or* .01 ; $\frac{1}{1000}$ *or* .001.

4. (*a*) 172.448 m. (*b*) 862.05 m. ; .00083 m. (*c*) 365.6 m. ; 391.948 ... m.

5. $272.19 ($272.1875). 6. $1801325 = $25000 × 72.053.

7. 7.64 mi. = 12.22 × $\frac{5}{8}$ mi. 8. 4800 km. (8 km. = 5 mi.).

9. $13\frac{1}{3}$ m. per sec. 10. $\frac{3}{1000}$.

11. 83711.7 m. = 83.7117 km. = (115020.7 − 31309) m.

12. 257 fr. (.75 m. @ 4 fr. + .625 m. @ 8 fr. + 6 m. @ 12 fr. + 7.08 m. @ 25 fr.).

Exercise 188. Page 208

1. 12000000 sq. m. 2. 8.35 sq. m.

3. $155163.648. 101.018 ares left. (He had 505.09 ares; he sold .8 of this, or 404.072 ares, @ $384 an are).

4. 12.4575 sq. m. 5. 2.92 sq. m. 6. 1000000 sq. m.

7. 2352.25 sq. m. = (48.5 × 48.5) sq. m.

8. 110.152 sq. m. = (15.68 × 7.025) sq. m.

9. $1055.60. (16.24 m. long and 7.85 m. wide: No. strips = $\frac{7.85}{.628} = 13$). 10. 28 m. 8 dm. (Area = 43.20 ares = 4320 centares = 4320 sq. m.). 11. 100 m.

12. $4650000 = \frac{2480000 \times 240}{16 \times 8}$.

Exercise 189. Page 209

1. .00514 cu. m. 2. 8.765345 cu. m.

3. 1219.24940201 cu. m. = 1219249.40201 cu. dm.

4. 175.5 steres. 5. $35.75. (27.5 Hl. = 275 Dl.).

6. 907.2 l. 7. 750 l. 8. $1.008 = 6c. × 2.4 × 7.

9. 809.6 cu. dm. 10. 100 phials.

Exercise 190. Page 210

2. 84 l. 3. 17.54 l. 4. 2c.
5. $10.55 ($10.546875). 6. 1000.
7. $4\frac{3}{8}$ gal., nearly. (4 l. = 7 pt.).
8. $3 67 ... gain. (10 Hl. = 1000 l. = $\frac{1762}{64}$ bu.).
9. 28210 l. 10. 1136 Hl.

Exercise 191. Page 211

1. 7018 g. 2. 2.77 g. 3. 12.021 g.
4. $72. (30 × 400 Kg. = 12000 Kg. = 12 tons). 5. 31 grains.
6. 2.179 ... lb. Avoir. (28 grams = 1 oz. Avoir.).
7. $4459\frac{11}{31}$ grams; $5419\frac{11}{31}$ grams. (1 lb. Troy = 5760 grains; 1 gram = $15\frac{1}{2}$ grains; 1 lb. Avoir. = 7000 grains).
8. $320\frac{5}{16}$ grains, if 28 grams = 1 oz.; $321\frac{1}{4}$ grains, if 1 gram = $15\frac{1}{2}$ grains.
9. 1.039 ... Kg. per sq. cm. {$2\frac{1}{5}$ lb. = 1 Kg.; ∴ $14\frac{3}{4}$ lb. = $\frac{295}{44}$ Kg. 1 m. = 39.37 in.; ∴ 1 cm. = .3937 in.; ∴ 1 sq. cm. = $(.3937)^2$ sq. in. = .155 sq. in., nearly. If pressure on 1 sq. in. = $\frac{295}{44}$ Kg., pressure on 1 sq. cm. = .155 × $\frac{295}{44}$ Kg.}. 10. 2500 men.

Exercise 192. Page 211

1. 900 sq. dm. 2. 3000 sq. cm. 3. 28.8 sq. dm.
4. 18 m. long; 12 m. broad; area is 216 sq. m.
5. 1.1382 ac. {Area = (72.5 × 62.8) sq. m. = 4553 sq. m. = .4553 Ha. = ($2\frac{1}{2}$ × .4553) acres}.
6. 47.61 Km. 7. 80 l. for each person. 8. 74 dm.
9. $11\frac{1}{4}$ dm. $\left(1080 \text{ Hl.} = 108000 \text{ l.} \quad \text{Depth} = \frac{108000}{120 \times 80} \text{ dm.}\right)$.
10. (*a*) 47.007008 cu. m.; (*b*) 57.019084 cu. m.; (*c*) 758.000078 cu. m.
11. 8000 cu. m. (Area of one face = 400 sq. m.; ∴ edge of cube = 20 m.).
12. 1.085 Dl.

Exercise 193. Page 212

1. 6.2137 mi. (10 Km = 393700 in.; 1 mi. = 63360 in.).
2. \$153.
3. \$64.94 = $\$1\frac{13}{17}$ × 36.8.
4. 61800 = 278100 ÷ $4\frac{1}{2}$.
5. $27\frac{7}{8}$ cm. (1 l. = 1 c.d.m. = 1000 c.c.).
6. 302400 Kg. (1 c. dm., or 1 l., water weighs 1 Kg.).
7. 38.4 Kg.
8. $1\frac{3}{4}$ times.
9. 1400 nickels.
10. 119.68 cu. m.

Exercise 194. Page 213

1. Rect. field is 450 m. long and 150 m. wide: ∴ area = 67500 sq. m.; area of sq. field = 62500 sq. m.; ∴ Rect. field = $\frac{27}{25}$ sq. field.
2. 1.2375 acres. (Area = 4950 sq. m. = 49.50 ares = .495 Ha. = $2\frac{1}{2}$ × .495 acres).
3. Circum. = 1166 cm. = 11.66 m. Area = 108146.5 sq. cm. = 10.81465 sq. m.
4. \$125.71$\frac{3}{7}$.
5. (*a*) 24700 cu. dm.; (*b*) 18476.89 m.; (*c*) 87000 dg.; (*d*) 8400 sq.m.; (*e*) 580000 cg.; (*f*) 490000 sq. cm.
6. 19.68 l.
7. 2.76 l. (1 l. water weighs 1 Kg.).
8. $189\frac{7}{17}$ = 805 ÷ 4.25.
9. 3.9 Kg. = (50 × 4 × 2.5 × 7.8) g.
10. 375 Kg. (1 cu. m. water weighs 1000 Kg.).
11. 337500 Kg.

EXERCISE XXXVII. PAGE 184

6. Representing B's share by 1,
A's " will be 3,
C's " " " 4.
∴ all the shares = 8 times B's "

7. When the second receives 8 apples, the first receives 7, and the third 10.

9. $2\ A\text{'s} + 2\ B\text{'s} + 2\ C\text{'s}$ do $\frac{46}{20}$ of work.
∴ $A + B + C$ " $\frac{23}{20}$ "
∴ A does $\frac{10}{20}$; B, $\frac{5}{20}$; and C, $\frac{8}{20}$ of work.

11. Take the first part as the unit, then the second part will be $\frac{3}{5}$ of the first, and the third will be $\frac{3}{8}$ of the first.
Sum of the parts $= (1 + \frac{3}{5} + \frac{3}{8})$ of first $= \frac{79}{40}$ of first part.

17. The pay of 7 women = the pay of 3 men.
" " " 14 boys = " " " $\frac{28}{5}$ women
= " " " $\frac{12}{5}$ men.
5 men + 3 men + $\frac{12}{5}$ men = $\frac{52}{5}$ men.

$$\text{Share of the men} = \frac{5}{\frac{52}{5}} \text{ of } \$10.40 = \$5.$$

18. Since there are 9 women, there must be 6 men and 15 children.
But the share of 9 women = share of 6 men,
and " " " 15 children = " " 5 "

20. Take B's share as the unit,
then C's " $= \frac{2}{5}$ of B's + \$800,
then A's " $= \frac{5}{9}$ " ($\frac{2}{5}$ of B's + \$800) − \$300.
Sum of all the shares $= \frac{73}{45}$ " B's + \$944$\frac{4}{9}$.

26. 2% of A's capital = \$220;
∴ " " $= \$\frac{100 \times 220}{2} = \$11000$;
∴ B's and C's " $= \frac{3}{2}$ of \$11000 = \$16500.

27. The shares are in the ratio of 1 × 4 × 10,
2 × 3 × 12, and 3 × 1 × 20.

30. 20 men = 30 women, and 50 children = 30 women;
∴ sum received by (30 + 40 + 30) women
= $600 for 1 week's work.

33. The ratio of costs is as 2 × 9 × 25 × 12 to 1 × 8 × 18 × $10\frac{1}{2}$.

35. Share of first = $\frac{3}{10}$ of $300 = $90
" " second = $\frac{3}{10}$ " $100 = $30.

But as the machine works the same length of time for each and earns $120 in all, or $60 for each, therefore, the latter must give the former the difference between $60 and $30, or $30.

36. Value of $10\frac{1}{2}$ oz. gold and $10\frac{1}{2}$ oz. silver = $49 + $(49 + 91)
= $189
Value of $10\frac{1}{2}$ oz. gold = $168;
∴ " $10\frac{1}{2}$ " silver = $21.

40. Weight of nitre = $(\frac{75}{100} + \frac{77}{100})$ of 10 cwt. = $15\frac{1}{5}$ cwt.

41. 4285 − (2540 + 980) = 765, the number of grains of soda and potash that take up 980 grains of the sulphuric acid;

$$\text{hence, } \frac{49 \times \text{number of grains of soda}}{32} + \frac{49 \times (765 - \text{number of grains of soda})}{48} = 980;$$

or, 3 × number of grains of soda + 2 × (765 − number of grains of soda) = 20 × 96;
∴ number of grains of soda = 1920 − 1530 = 390;
∴ " " " " potash = 765 − 390 = 375.

42. 15 men and 30 children get £(177 − 60), or £117;
∴ 1 man " 2 " " £$\frac{117}{15}$, or £7 16*s*.
But 1 " " 1 child " £6;
∴ 1 " " £1 16*s*.

EXERCISE XXXVIII. PAGE 192

6. Sum to be divided between *A* and *B* = $\frac{45}{64}$ of $29000.
A's share = $\frac{128}{225}$ of $\frac{45}{64}$ of $29000 = $11600.

7. *A*'s money was in the business 287 days, and *B*'s 167 days

$$\begin{aligned} 287 \times 2400 &= 688800 \\ 167 \times 1800 &= 300600 \\ & 989400; \end{aligned}$$

$\therefore$ *A*'s share $= \frac{688800}{989400}$ of \$943 $=$ \$656$\frac{820}{1649}$.

11. $2000 + 2800 + 1686 + 1014 = 7500$;

$\therefore$ *D*'s share of capital $= \frac{1014}{7500}$ of \$45000 $=$ \$6084.

12. *A* and *B*, and *A* and *C* contribute \$(1390 + 1500), or \$2890;
B and *C* contribute \$1590;

$\therefore$ twice *A*'s contribution $=$ \$1300;

hence *A* contributes \$650, *B* \$740, *C* \$850, *D* \$960.

14. \$960 gains \$240 in 6 mo., or \$480 in 12 mo.;

$\therefore$ \$100 gains \$50 in 12 mo.

\$150 is amount of \$100 capital.
\$2400 " " " \$1600 "
\$50 is gained on \$100 in 12 mo.
\$400 " " " \$640 " $\frac{12}{1} \times \frac{100}{50} \times \frac{400}{640}$ mo.

16.

$$\left.\begin{aligned} 12 \times 400 &= 4800 \\ 10 \times 500 &= 5000 \end{aligned}\right\} \; 9800 = A\text{'s capital for 1 mo.}$$

$$\left.\begin{aligned} 12 \times 300 &= 3600 \\ 9 \times 600 &= 5400 \end{aligned}\right\} \; \underline{9000 = B\text{'s} \quad " \quad " \quad " \quad "}$$

18800.

A's share $= \frac{9800}{18800}$ of \$470 $=$ \$245.

18.

$$\left.\begin{aligned} 4 \times 1000 &= 4000 \\ 5 \times 2500 &= 12500 \\ 3 \times 2200 &= 6600 \end{aligned}\right\} \; 23100 = A\text{'s capital for 1 mo.}$$

$$\left.\begin{aligned} 6 \times 2500 &= 15000 \\ 5 \times 3100 &= 15500 \\ 1 \times 4100 &= 4100 \end{aligned}\right\} \; 34600 = B\text{'s} \quad " \quad " \quad "$$

$$\left.\begin{aligned} 4 \times 3000 &= 12000 \\ 4 \times 1000 &= 4000 \\ 4 \times 500 &= 2000 \end{aligned}\right\} \; 18000 = C\text{'s} \quad " \quad " \quad "$$

19.
$$\left.\begin{array}{l}1\tfrac{1}{2}\times\ \ 10=\ \ 15 \text{ horses}=\ \ 45 \text{ sheep}\\ 2\ \ \times\ \ 30=\ \ 60 \text{ oxen}\ \ =120\ \ \text{“}\\ 3\tfrac{1}{4}\times100=325 \text{ sheep}=325\ \ \text{“}\end{array}\right\}=490.$$
$$\left.\begin{array}{l}2\tfrac{1}{2}\times\ \ 40=100 \text{ horses}=300 \text{ sheep}\\ 1\tfrac{1}{4}\times\ \ 50=62\tfrac{1}{2} \text{ oxen}\ \ =125\ \ \text{“}\\ 3\ \ \times115=345 \text{ sheep}=345\ \ \text{“}\end{array}\right\}=770.$$

22. *A* uses the *whole* house for 4 mo.; half of it for $5\frac{1}{2}$ mo., and $\frac{1}{3}$ of it for $2\frac{1}{2}$ mo.

B uses $\frac{1}{2}$ the house for $5\frac{1}{2}$ mo., and $\frac{1}{3}$ of it for $2\frac{1}{2}$ mo.

C uses $\frac{1}{3}$ the house for $2\frac{1}{2}$ mo.

Exercise XXXIX. Page 201

6.
$$£1=\frac{109\frac{5}{8}}{100} \text{ of } \$4\tfrac{4}{9}=\tfrac{877}{800} \text{ of } \$\tfrac{40}{9};$$

$$\text{Cost of draft for } £1=\frac{100\frac{1}{4}}{100} \text{ of } \tfrac{877}{800} \text{ of } \$\tfrac{40}{9}.$$

7.
$$\text{Since } £1500=\$7300,$$
$$\therefore\ \ £1=\$4\tfrac{13}{15}.$$

$$\text{Now the advance on } \$4\tfrac{4}{9}=\$(4\tfrac{13}{15}-4\tfrac{4}{9})=\$\tfrac{19}{45};$$
$$\therefore\ \ \text{“}\quad\text{“}\quad\text{“}\ \$100=\$\frac{100\times\frac{19}{45}}{4\frac{4}{9}}$$
$$=\$9\tfrac{1}{2}.$$

Hence, exchange is at a premium of $9\frac{1}{2}$%, and the quotation would be $109\frac{1}{2}$.

10. Interest on \$1 for 63 da. at 6% = \$.0103561 ;

∴ present worth of a draft for \$1 = \$.9896439 ;

$$\therefore \text{ cost of draft for } \$1=(\$.9896439+.0025)$$
$$=\$.9921439.$$

20. The course of exchange is quoted by giving the value of 4 marks in cents.

Hence, the course of exchange is 4 marks = 95c.

EXERCISE XL. PAGE 203

10. The reciprocal ratio is expressed by inverting the order of the terms; hence, the reciprocal ratio of $\frac{1}{3} : \frac{1}{6}$ is $\frac{1}{6} : \frac{1}{3}$, or $\frac{1}{6} \div \frac{1}{3}$. $\frac{1}{6} : \frac{1}{3} = \frac{1}{2}$.

Again, the reciprocal ratio of $2\frac{1}{3} : 7.\dot{7}$ is $7.\dot{7} : 2\frac{1}{3}$, or $7\frac{7}{9} \div 2\frac{1}{3}$. $7\frac{7}{9} \div 2\frac{1}{3} = 3\frac{1}{3}$.

13. If $\frac{2}{3}$ be the given ratio, then adding any number, say 5, to each of the terms, we have $\frac{2+5}{3+5} = \frac{7}{8}$. Comparing this ratio with $\frac{2}{3}$, we have $\frac{16}{24}$ and $\frac{21}{24}$. Hence, we see that this ratio is increased by adding the same number to each of its terms.

Again, if we take $\frac{4}{3}$ as the ratio and add, say 2, to each of the terms, we have $\frac{4+2}{3+2} = \frac{6}{5}$. Comparing this with the original ratio, we have $\frac{20}{15}$ and $\frac{18}{15}$. Hence, we see that this ratio is diminished by adding the same number to each of its terms. A ratio is, therefore, increased or diminished by adding the same number to each of its terms according as the antecedent is less or greater than its consequent.

EXERCISE XLI. PAGE 206

11. The proportion is 30 ft. : $12\frac{1}{2}$ ft. :: \$$x$: \$650 ;

$$\therefore \ \$x = \$\frac{30 \times 650}{12\frac{1}{2}} = \$1560.$$

18. 84 ft. : $2\frac{1}{4}$ ft. :: 125 ft. : height required ;

$$\therefore \text{ height required} = \frac{2\frac{1}{4} \times 125}{84} \text{ ft.} = 3\tfrac{39}{112} \text{ ft.}$$

28. The 5000 men have provisions for 90 da. remaining.
Hence, 4500 men : 5000 men :: 90 da. : time required;

$$\therefore \text{ time required} = \frac{5000 \times 90}{4500} \text{ da.} = 100 \text{ da.}$$

Hence, they will last 10 da. longer.

EXERCISE XLII. PAGE 212

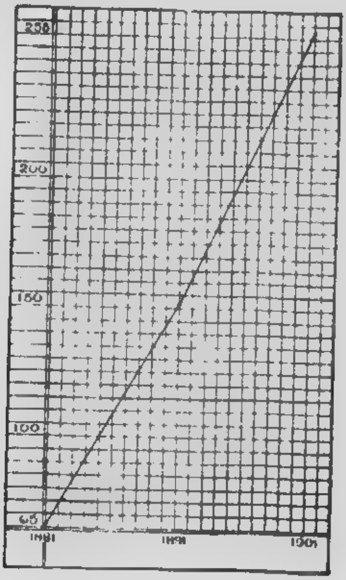
1. Draw two lines, one 5 dm. and the other 8 dm. in length The former will represent the kilometre and the latter the mile.

2. Draw two lines, one 5 dm. and the other 11 dm. in length. The former will represent the pound, and the latter the kilogram

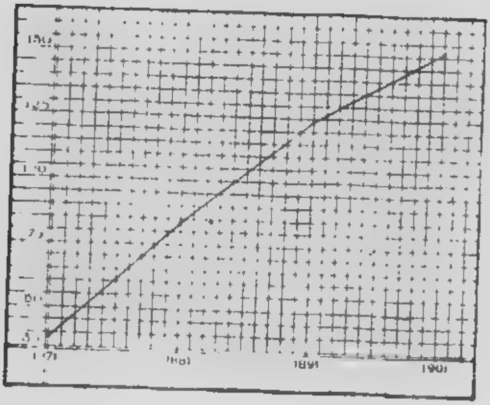
3. Draw two lines, one 25 cm. and the other 44 cm. in length Since 25 l. = 44 pt., the latter will represent the litre and the former the pint.

4. Draw two lines, one 37 cm. and the other 10 cm. in length The former will represent the cord and the latter the stere.

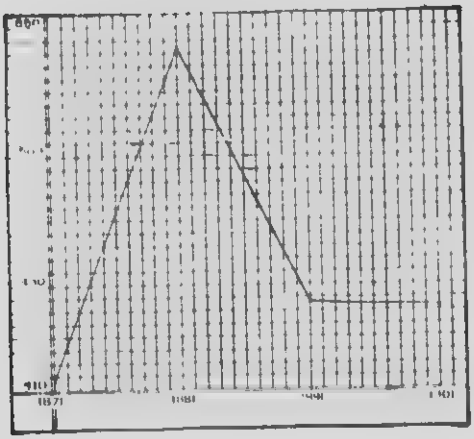
5.



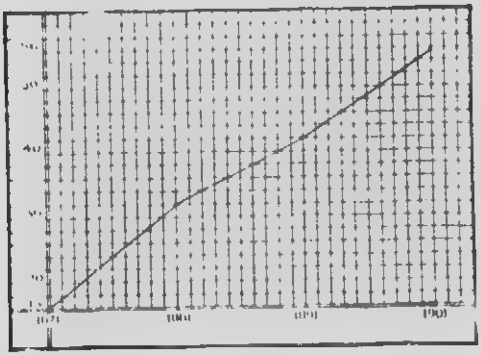
6.



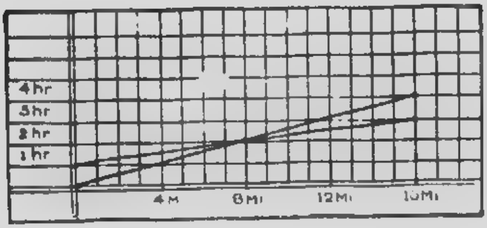
7.



8.

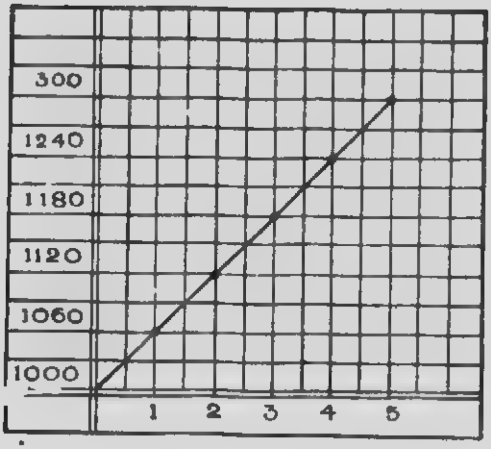


10.

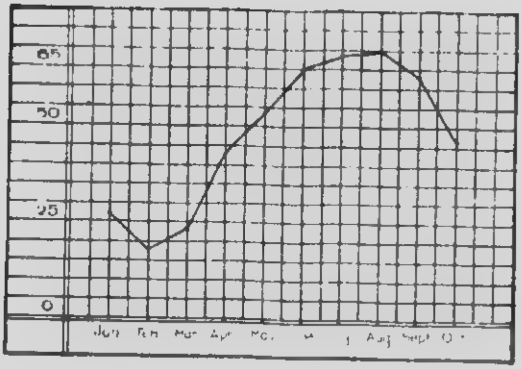


The graph shows that Jones' rate is 8 mi. per hour and that he overtakes Smith at the end of 8 mi.

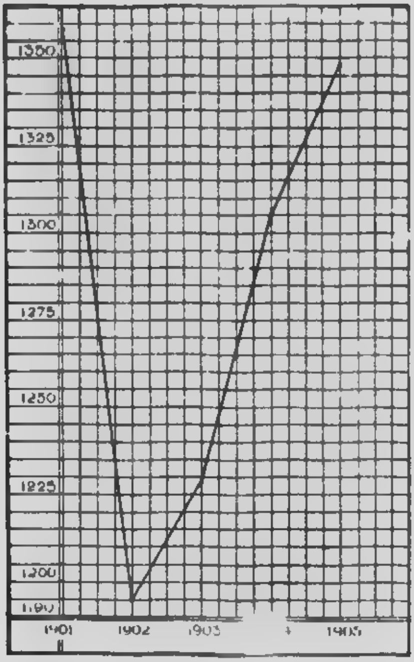
11


300
1240
1180
1120
1060
1000
1
2
3
4
5

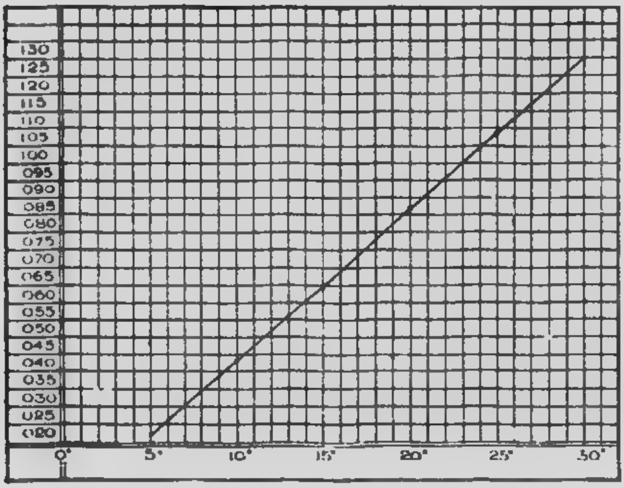
12.


65
50
25
0
Jan
Feb
Mar
Apr
Aug
Sept

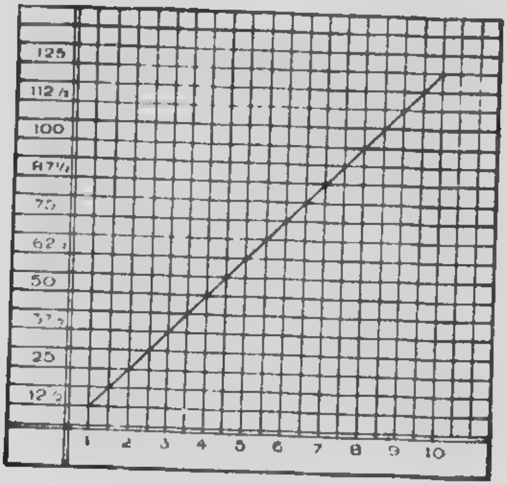
13.


1350
1325
1300
1275
1250
1225
1200
1190
1901
1902
1903
4
1905

14.

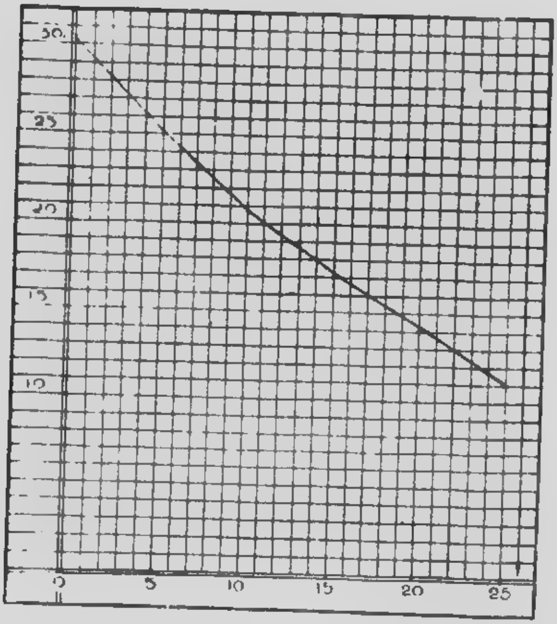

130
125
120
115
110
105
100
095
090
085
080
075
070
065
060
055
050
045
040
035
030
025
020
0°
5°
10°
15°
20°
25°
30°

15.



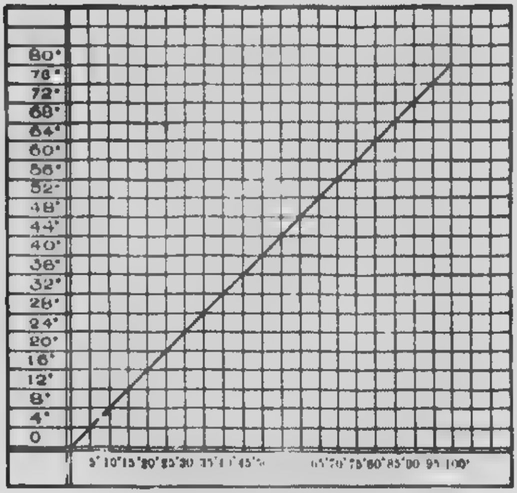
NUMBER OF YARDS.

16.



From the graph it will be seen that the barometer will be 27.1 in high at 3000 ft.; 28.2 in. at 2000 ft. and 22.5 at 8000 ft.

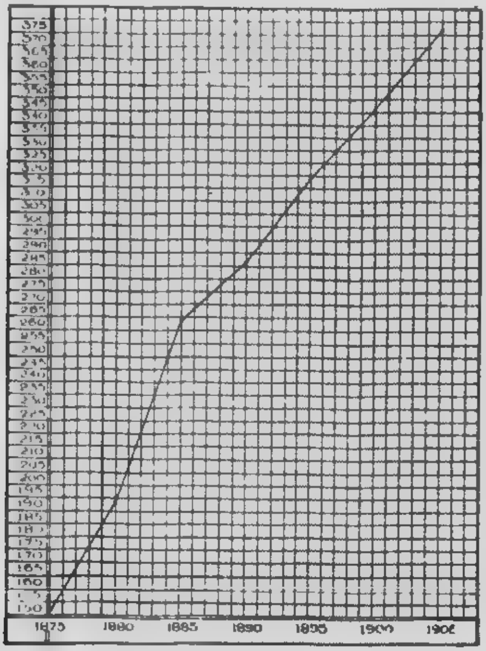
18.



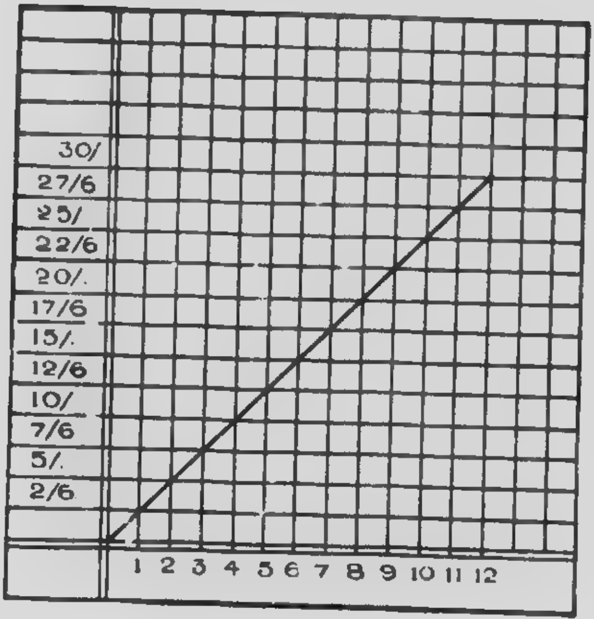
DEGREES CENTIGRADE.

It is easily seen that 10 R. is equal to 12½° C., and that 15° C. is equal to 12° R.

19.

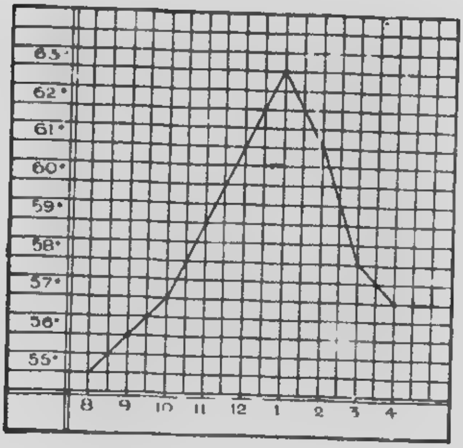


21.



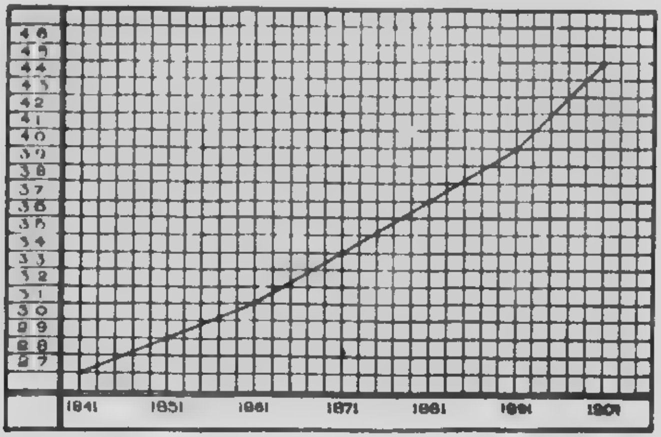
NUMBER OF POUNDS.

22.



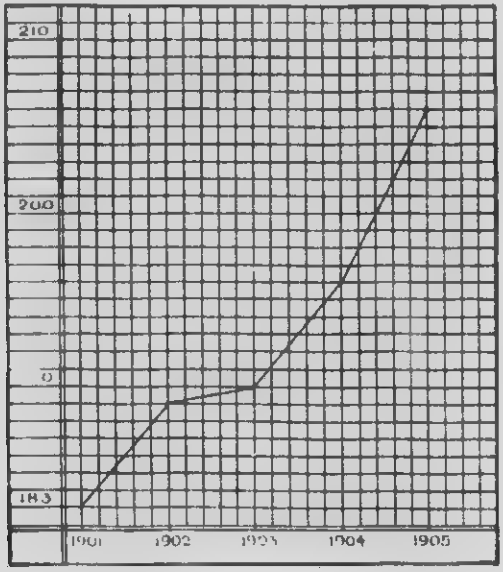
The following is the graph for Scotland.

23.



(*c*) Draw the graph for the combined populations.

24. (*a*)



EXERCISE XLIII. PAGE 219

5. Side of square $= \frac{160}{4}$ in. $= 40$ in.

Sides of rectangle $= \frac{2}{5}$ of 80 in. and $\frac{3}{5}$ of 80 in.

Area of square $= 40^2$ sq. in. $= 1600$ sq. in.

Area of rectangle $= (32 \times 48)$ " " $= 1536$ " "

$\therefore$ difference of area $= 64$ sq. in.

EXERCISE XLVII. PAGE 226

4. Distance of top from ground $= \sqrt{(41^2 - 9^2)}$ ft. $= 40$ ft.

$\therefore$ distance the top will drop $= (41 - 40)$ " $= 1$ ft.

11 Let $d =$ distance from the ground.

Then $(98 - d)^2 = d^2 + 84^2$;

$\therefore 9604 - 196d + d^2 = d^2 + 7056$;

$\therefore 196d = 2548$;

$\therefore d = \frac{2548}{196} = 13$;

$\therefore$ the tree broke 13 ft. from the ground.

EXERCISE XLVIII. PAGE 229

When the triangle is a right-angled one the length of the perpendicular dropped on the hypothenuse is most readily found as follows:

Since twice the measure of the area of the triangle is equal to the product of the measures of the sides containing the right angle and is also equal to the product of the measures of the perpendicular on the hypothenuse and the hypothenuse, therefore, to find the measure of this perpendicular divide the product of the measures of the sides containing the right angle by the measure of the hypothenuse.

EXERCISE XLIX. PAGE 230

2. The quadrilateral is divided by the diagonal into two triangles.

4. The side A D is (733 − 325) ft. longer than B C.

733 ft. − 325 ft. = 408 ft.

$\frac{408}{2}$ ft. = 204 "

Width of A B C D = $\sqrt{(325^2 - 204^2)}$ ft.

= 253 ft.

Area of A B C D = $\left(\frac{733+325}{2} \times 253\right)$ sq. ft.

= 133837 sq. ft.

Exercise LI. Page 233

7. The diagonal of the square is the diameter of the circle.

Now diameter of circle = $\frac{7}{22}$ of 88 in. = 28 in.

Hence, 28 in. is the hypothenuse of a right-angled triangle, the other sides being the sides of the square;

hence, $2 \times (\text{measure of side})^2 = 28^2$;

$\therefore (\text{measure of side})^2 = \frac{784}{2} = 392$;

$\therefore$ measure of side = $\sqrt{392}$

= 19.79;

$\therefore$ side of square = 19.79 in.

8. Radius of outer circle = $\frac{1}{2}$ of $\frac{7}{22}$ of 880 yd. = 140 yd.

" " inner " = $\frac{1}{2}$ " $\frac{7}{22}$ " 792 " = 126 "

$\therefore$ width of road = 14 yd.

Area of outer circle = $(\frac{22}{7} \times 140^2)$ sq. yd.

" " inner " = $(\frac{22}{7} \times 126^2)$ " "

$\therefore$ area of road = $(\frac{22}{7} \times 266 \times 14)$ sq. yd.

= 11704 sq. yd.

Exercise LII. Page 235

4. Radius of circle = $\sqrt{(\frac{7}{22} \text{ of } 616)}$ ft. = 14 ft.

Now, measure of length of arc × 14 = 2 × 56;

$\therefore$ measure of length of arc = $\frac{2 \times 56}{14} = 8$;

$\therefore$ " " " = 8 ft.

EXERCISE LIII. PAGE 236

10. Draw the figure and let the produced sides meet in a, and let the distance between a and the shorter parallel side be x and y respectively; then there are two similar triangles formed and

$$18 : 16 + x :: 10\tfrac{1}{2} : x \ (1);$$
$$\text{also, } 18 : 12 + y :: 10\tfrac{1}{2} : y \ (2).$$

Solving (1), x is found to be $22\tfrac{2}{5}$, and
" (2), y " " $16\tfrac{4}{5}$;

$\therefore$ distance required $= (22\tfrac{2}{5} + 16)$ ft. $= 38\tfrac{2}{5}$ ft.
and " " $= (12 + 16\tfrac{4}{5})$ " $= 28\tfrac{4}{5}$ "

13. The diameters of the pipes are as 2 : 1.
The areas of the bores " " " 4 : 1.

Area of a pipe to do as much work as the two would be represented by 5 in (a) and 3 in (b).

Now, time for a pipe whose bore is represented by 4 to fill the cistern = 20 min.;

$\therefore$ " " " " " represented by 5 to fill the cistern $= \dfrac{4 \times 20}{5}$ min.
$= 16$ min.,

and " " " " " represented by 3 to fill the cistern $= \dfrac{4 \times 20}{3}$ min.
$= 26\tfrac{2}{3}$ min.

EXERCISE LVI. PAGE 248

8. Area of surface of ring $= (\frac{22}{7} \times 33^2 - \frac{22}{7} \times 30^2)$ sq. in.
$= (\frac{22}{7} \times 63 \times 3)$ sq. in.
$= 594$ sq. in.;

$\therefore$ volume of ring $= (2 \times 594)$ cu. in.
$= 1188$ cu. in.

14. Cubic content of vessel $= (4 \times 277.274)$ cu. in.
If the radius is r, then the cubic content in terms of

$$r \text{ is } \tfrac{22}{7} \times r^2 \times 2r;$$

$$\therefore \tfrac{22}{7} \times r^2 \times 2r = 4 \times 277.274;$$

$$\therefore r = \sqrt[3]{\left(\frac{4 \times 277.274}{\frac{22}{7} \times 2}\right)} \text{ in.}$$

$$= 5.6 \text{ in., nearly};$$

$$\therefore \text{depth} = 2 \times 5.6 \text{ in., nearly}$$

$$= 11.2 \text{ in.}$$

Exercise LVII. Page 250

4. Perimeter of base $= 5 \times 18$ in. $= 90$ in.

Area of lateral surface $= \dfrac{7\frac{1}{2} \times 10}{2}$ sq. ft. $= 37\frac{1}{2}$ sq. ft.

Exercise LVIII. Page 25[illegible]

6. Altitude of pyramid $= \sqrt{(61^2 - 11^2)}$ ft. $= 60$ ft.

Volume " " $= \dfrac{60 \times 22 \times 22}{3}$ cu. ft. $= 9680$ cu. ft.

Exercise LIX. Page 253

5. Radius of cone $= \frac{1}{2}$ of $\frac{7}{22}$ of 342 in. $= \frac{1197}{22}$ in.

Slant height $= \sqrt{\{(252)^2 + (\frac{1197}{22})^2\}}$ in. $= 257.806$ in.

Lateral surface $= (\frac{342}{2} \times 257.806)$ sq. in. $= 44084.826$ sq. in.

8. Slant height of vessel $= \sqrt{(70^2 + 24^2)}$ in. $= 74$ in.

Area of lateral surface $= (74 \times \frac{22}{7} \times \frac{48}{2})$ sq. in.

$= \frac{39072}{7}$ sq. in.

Area of lid $= (\frac{22}{7} \times 24 \times 24)$ sq. in.

$= \frac{12672}{7}$ sq. in.;

$\therefore$ number of square feet of iron $= \left(\dfrac{39072 + 12672}{7 \times 144}\right) = 51\frac{1}{3}$.

Exercise LXI. Page 256

8. Since 12 in. represents 8000 mi.,

1 " " $\frac{8000}{12}$ mi., or $666\frac{2}{3}$ mi.

EXERCISE LXII. PAGE 257

2. Cubic content of block = 1 cu. ft.

" " " sphere $= (\frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$ cu. ft.

$= \frac{11}{21}$ cu. ft.;

∴ part cut away $= \frac{10}{21}$ cu. ft.

$= 822\frac{6}{7}$ cu. in.

6. Cubic content of sphere $= (\frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6)$ cu. in.

" " " cylinder $= 8 \times \{\frac{22}{7} \times (6\frac{1}{2})^2 - \frac{22}{7} r^2\}$

$= 8 \times \frac{22}{7}\{(6\frac{1}{2})^2 - r^2\}$;

$\therefore\ 8 \times \frac{22}{7}\{(6\frac{1}{2})^2 - r^2\} = \frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6$;

$$(6\tfrac{1}{2})^2 - r^2 = \frac{\frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6}{8 \times \frac{22}{7}} = 36;$$

$\therefore\ r^2 = 6\frac{1}{4}$;

$\therefore\ r = \frac{5}{2} = 2\frac{1}{2}$.

Hence, the thickness of the iron $= (6\frac{1}{2} - 2\frac{1}{2})$ in. $= 4$ in.

7. Let the volume of each be 1 cu. ft.

Surface of cube $= 6 \times 1$ sq. ft.

$= 6$ sq. ft.;

Also, $\frac{4}{3} \times \frac{22}{7} \times r^3 = 1$;

$\therefore\ r = \sqrt[3]{(\frac{3}{4} \text{ of } \frac{7}{22} \times 1)}$ ft.;

∴ surface of sphere $= \{4 \times \frac{22}{7}(\sqrt[3]{\frac{21}{88}})^2\}$ sq. ft.

$= \{4 \times \frac{22}{7} \times \frac{1}{88} \times \sqrt[3]{21^2 \times 88}\}$ sq. ft.

$= (\frac{1}{7} \times 33.853 \ldots)$ sq. ft.

$= 4.833 \ldots$ sq. ft.;

∴ surface of cube : surface of sphere :: 6 : 4.833...

:: 1 : .805...

11. Surface of the lead ball $= (\frac{22}{7} \times 4 \times 4)$ sq. in.;

∴ " " " silver " $= (2 \times \frac{22}{7} \times 4 \times 4)$ sq. in.;

∴ diameter " " " " $= \sqrt{(\frac{7}{22} \times 2 \times \frac{22}{7} \times 4 \times 4)}$ in.

$= 5.6568 \ldots$ in.;

∴ thickness of the silver $= \left(\frac{5.6568 - 4}{2}\right)$

$= .8284$ in.

13. Cubic content of ball $= (\frac{1}{6} \text{ of } \frac{22}{7} \text{ of } 27)$ cu. in.

$= \frac{99}{7}$ cu. in.

Area of base of vessel $= (3\frac{1}{7} \times 3^2)$ sq. in.

$= \frac{198}{7}$ sq. in.

Height to which the water rises $= (\frac{99}{7} \div \frac{198}{7})$ in.

$= \frac{1}{2}$ in.

Hence, the water will be $4\frac{1}{2}$ in. deep in the vessel.

EXERCISE LXIII. PAGE 261

5. Cubic content $= \{\frac{1}{3} \times 25 \times (\frac{77}{2} + \frac{77}{128} + \frac{77}{16})\}$ cu. m. $= 365\frac{365}{384}$ cu. m.

6. Area of section of drain $= \left(\dfrac{25+16}{2} \times 4\right)$ sq. ft.

$= 82$ sq. ft.

Distance water flows in 60 min. $= 2640$ ft.;

$\therefore$ " " " " 10 " $= \frac{1}{6}$ of 2640 ft. $= 440$ ft.

Cubical content of drain in 440 ft. $= (440 \times 82)$ cu. ft.

Now, 1 cu. ft. of water $= 6\frac{1}{4}$ gal.;

$\therefore (440 \times 82)$ " " " " $= 440 \times 82 \times 6\frac{1}{4}$ gal.

$= 225500$ gal.

8. Altitude of frustum $= \sqrt{(5^2 - 3^2)}$ ft. $= 4$ ft.

EXERCISE LXIV. PAGE 263

3. If d and d' represent the diameters of a 25c. piece and a 5c. piece, respectively, then the areas of the circular faces are as d^2 to $(d')^2$, and the quantity of silver in each as $7d^2$ to $5(d')^2$;

$$\therefore\ 7d^2 = 5 \times 5(d')^2;$$
$$\therefore\ d\sqrt{7} = 5d';$$
$$\therefore\ d : d' :: 5 : \sqrt{7}.$$

4. Volume of first : $3 \times$ volume of first :: 20^3 : (measure of breadth of second)^3;

$\therefore$ (measure of breadth of second)$^3 = 3 \times 20^3$;

$\therefore$ measure of breadth of second $= 20\sqrt[3]{3}$

28.845;

$\therefore$ breadth of second $= 28.845$ ft.

10. $27 : 125 :: 1^3 : (\text{measure of dimension})^3$;
∴ measure of dimension $= \frac{5}{3}$.
Now area of first : area of second $:: 1^2 : (\frac{5}{3})^2$;
∴ area of second $= \frac{25}{9} \times$ area of first.
Again, cost of first = \$3.60;
∴ " " second $= \frac{25}{9} \times \$3.60 = \$10.00$.

13. Volume of first plate $= (\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 1)$ cu. in.
$= 38\frac{1}{2}$ cu. in.
" " second " $= (\frac{22}{7} \times 7 \times 7 \times 1)$ cu. in.
$= 154$ cu. in.
Volume of both plates $= 192\frac{1}{2}$ cu. in.;
∴ area of circular plate $= \frac{192\frac{1}{2}}{5}$ sq. in. $= 38.5$ sq. in.
Now, the radius of plate $= \sqrt{(\frac{7}{22} \text{ of } 38.5)}$ in. $= \frac{7}{2}$ in.;
∴ diameter of plate = 7 in.

Exercise LXV. Page 264

3. Measure of diagonal of cube = measure of edge $\times \sqrt{3} = 6$;
∴ measure of edge $\times \sqrt{3} = 6$;
∴ measure of edge $= \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$;
∴ volume of cube $= (2\sqrt{3})^3$ cu. ft. $= 41.568$ cu. ft.

10 The volumes will vary as the areas of the similar surfaces;
∴ area of first plate : area of part cut away $:: 24^2 : (\text{diameter of hole})^2$;
∴ $1 : \frac{1}{4} :: 24^2 : (\text{diameter of hole})^2$;
∴ diameter of hole $= \sqrt{(\frac{1}{4} \times 24^2)}$ in. $= 12$ in.

12. Volume of bottom $= 1 \times (72 \times 42 \times 1)$ cu. in.
$= 3024$ cu. in.
" " sides $= 2 \times (47 \times 72 \times 1)$ cu. in.
$= 6768$ cu. in.
" " ends $= 2 \times (40 \times 47 \times 1)$ cu. in.
$= 3760$ cu. in.;
∴ volume of iron $= 13552$ cu. in.

The following is a simpler solution :

Volume, if the box were solid $= (72 \times 42 \times 48)$ cu. in. $= 145152$ cu. in.

" of the interior space $= (70 \times 40 \times 47)$ cu. in. $= 131600$ cu. in.

$\therefore$ volume of iron $= 13552$ cu. in.

13. Number of cubic feet of water in the cistern $= \dfrac{93.75 \times 112 \times 16}{1000}$

15. The cubic content of a brick in the second case $= (\frac{4}{5})^3$ of that of a brick in the first case.

Hence, we may leave the exact dimensions of the first brick out of account and find the cost thus:—

$$\text{cost of 1 brick} = \$\left(\tfrac{64}{125} \text{ of } \frac{213.50}{9760}\right);$$

$$\therefore \text{cost of 100 bricks} = \$\left(100 \times \tfrac{64}{125} \times \frac{213.50}{9760}\right) = \$1.12.$$

EXERCISE LXVII. PAGE 270

No. 1. (*a*)

Vol. of contents of cistern $= (8 \times 7 \times 6)$ cu. ft. $= 336$ cu. ft.

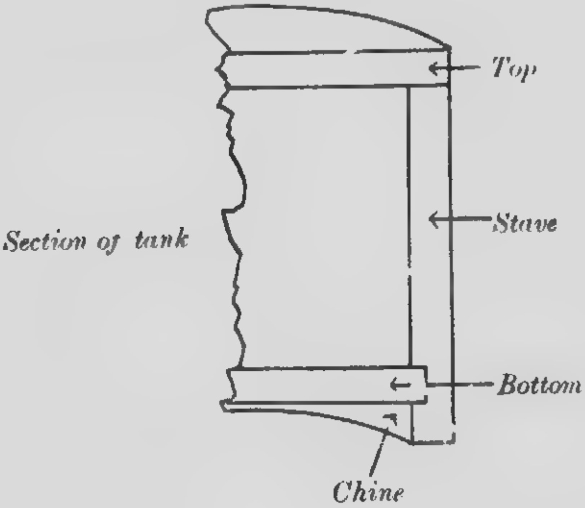
" of cistern $= (8\frac{1}{3} \times 7\frac{1}{3} \times 6\frac{1}{3})$ cu. ft. $= 387\frac{1}{27}$ cu. ft.

" of material $= (387\frac{1}{27} - 336)$ cu. ft. $= 51\frac{1}{27}$ cu. ft.

1 cu. ft. of wood $= 12$ board ft.;

$\therefore 51\frac{1}{27}$ cu. ft. $= 51\frac{1}{27} \times 12$ board ft. $612\frac{4}{9}$ board ft.

2.

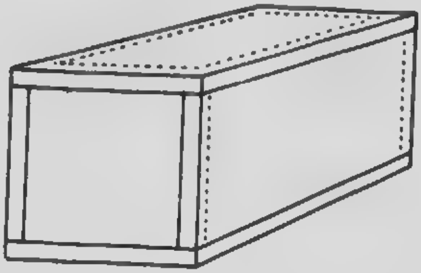


Section of tank

Circular tanks have a "*chine*" about equal to the thickness of bottom, which is rabbeted into the staves about $\frac{1}{2}$".

(*b*) Top, 115.5 b. ft.; bottom, 102.1 b. ft.; sides, 643.2 b. ft.

6. In box-work the sides are nailed on the *ends*, then top and bottom on sides and ends.



Top, $\frac{2 \times 30 \times 22}{144}$;

Sides, $\frac{2 \times 30 \times 16\frac{1}{4}}{144}$;

Ends, $\frac{2 \times 22\frac{1}{4} \times 16\frac{1}{4}}{144}$.

7. All common lumber and all squared stock are cut in 2" widths and 2' lengths; any fraction of these measurements requires the next larger size.

In sawing squared timbers into boards, about $\frac{1}{8}$" to $\frac{3}{16}$" is lost in sawdust in each saw kerf, depending on the way sawn, the number of kerfs is *one* less than the number of boards.

EXERCISE LXVIII. PAGE 272

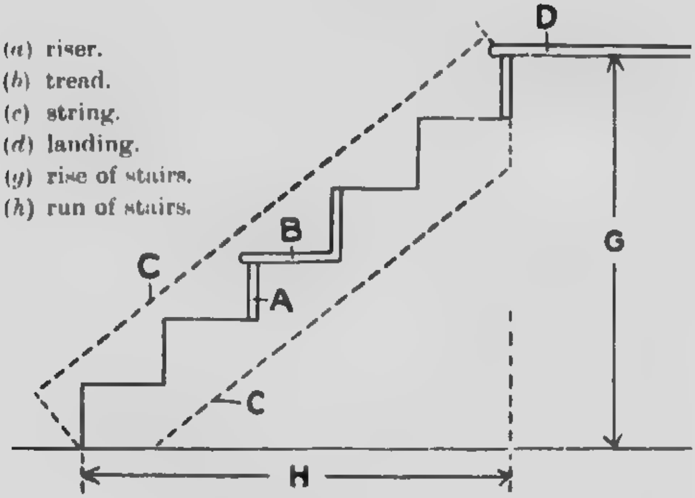
6. Unless otherwise stated, allow 25% for matching and cutting.

EXERCISE LXIX. PAGE 274

A stairs" consists of "STEPS" supported by "strings" or side pieces.

Each STEP consists of a "*riser*" and a "*tread*," except the top one, where the landing takes the place of the tread. There is thus one tread less than the number of risers.

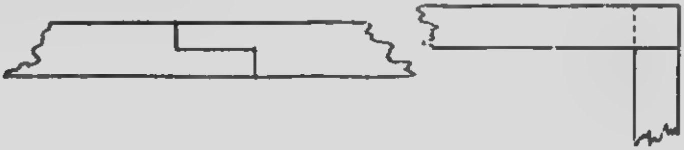
Stairs should be laid out so that twice the rise of a step + the tread = 24".



(*a*) riser.
(*b*) tread.
(*c*) string.
(*d*) landing.
(*g*) rise of stairs.
(*h*) run of stairs.

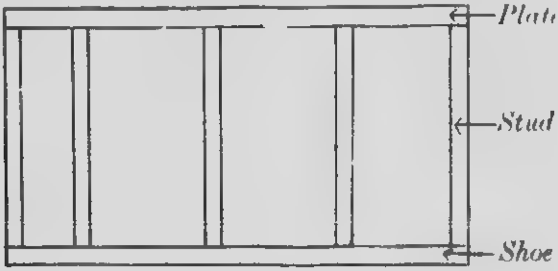
EXERCISE LXX. PAGE 275

1. Sills run full length and width of the building and are spliced with a lap joint.



In spacing for joists, studs, or rafters, one is placed on outside and others spaced from it, ending with one regardless of last space.

The studs in walls and partitions are held in place by a "*shoe*" and a "*plate*."



5. Read the following for this example: A frame house is 18 ft. wide by 24 ft. long. How many pieces of studding 3″ × 4″ placed 16″ from centre to centre are required for the outer walls, add 12 extra studs for double corners and double studs at windows and doors? If they are 18 ft. long, find their cost at $30 per m.

Ans.: 76 studs; $27.36.

Exercise LXXII. Page 277

1. Shingles are calculated at 4″ in width. Allowing for waste, it will require 1000 shingles to cover one square if laid 4″ to the weather, 900 shingles if laid 4½″ to the weather, 800 shingles if laid 5″ to the weather.

No allowance is made for waste in fancy shingles, which are usually cut 4″ in width. Extra allowance is then made for the double row at the eaves.

6. For this example read, "A round tower is 14 ft. in diameter, outside measurement. The roof is flat and the steel rafters project 2 ft. Find the cost of a galvanized iron roof at 12c. per square foot."

Exercise LXXIII. Page 278

1. Standard brick are laid with mortar so that in a wall 2 bricks thick (9″) it requires 14 bricks per square foot of surface; in a wall 3 bricks thick (13″) it requires 21 bricks per square foot of surface.

Exercise LXXIV. Page 280

In 7, 8, 9, follow actual practice regarding the openings.

Exercise LXXV. Page 283

2. (*f*) For 25 cm. read 6 cm. and for yard read metre.
There will be 15 widths of carpet required.

3. The hall is 4 yd. by 5 yd.
The kitchen is 5 yd. by 6 yd.

EXERCISE LXXVII. PAGE 286

1. (*c*) Read 12c. a yard.
   (*d*) Read 15c. a yard.

EXERCISE LXXVIII. PAGE 287

4. The length of the trench is $(100 + 40 - 6) \times 2$ ft.

6. Surface area of quarter section

   $= (160 \times 160 \times \frac{121}{4} \times \frac{9}{1} \times \frac{144}{1})$ sq. in.

   Cubic content of water supplied

   $= (160 \times 160 \times \frac{121}{4} \times \frac{9}{1} \times \frac{144}{1} \times 1)$ cu. in.

   This water would fill a section of the irrigation ditch $(7 \times 24 \times 2 \times 5280 \times 12)$ in. long.

   $$\therefore \text{ Area of cross section} = \frac{160 \times 160 \times 121 \times 9 \times 144}{4 \times 7 \times 24 \times 2 \times 5280 \times 12} \text{ sq. in.}$$

8. Perimeter = twice length + twice width

   = 6 widths.

9. (*a*) $21\frac{1}{3}$ bbls. are used ; 22 bbls. must be purchased.
   (*b*) The contractor would pay for 21 cu. yd. of concrete.

12. 2 sides each 4 ft. by 1 ft. 6 in.
    2 ends each 2 ft. 6 in. by 1 ft. 6 in.
    Top and bottom each 4 ft. 2 in. by 2 ft. 6 in.

The following is another solution :

Cubic content of space occupied by box
$= (50 \times 30 \times 20)$ cu. inches $= 30000$ cu. in.

Cubic content of space inside of box
$= (48 \times 24 \times 18)$ cu. in. $= 24192$ cu. in.;

$\therefore$ cubic content of lumber $= 5808$ cu. in.
$= 40\frac{1}{3}$ board ft.

13. Area is $\sqrt{s(s-a)(s-b)(s-c)}$ where a, b, c are the sides of the triangle and s is one-half their sum.

15. $85^2 + 132^2 = 157^2$, hence the triangle is right-angled.

Exercise LXXIX. Page 289

8. Assume that the field is a square. One side will be 40 rods. There will be 220 rows and 220 hills in a row.

12. The answer given supposes the field to be a square. It will contain 16 rows of trees and 16 trees in a row.

15. In answering this question the best plan is to apply enough farm manure to return the full amount of phosphoric acid which has been taken out of the soil by the timothy. This would mean a six-ton application. In doing this an additional 20 lbs. of nitrogen and 54 lbs. of potash are required, in order to balance up the soil account. To supply the balance of 20 lbs. of nitrogen and the 54 lbs. of potash an additional problem should be introduced. Sodium nitrate contains 15.6% of nitrogen and a sufficient quantity of this material could be purchased to reinforce the nitrogen found in the manure. Potassium sulphate contains 48% of potash. How much sodium nitrate and how much potassium sulphate should be used to meet the above deficiencies?

16. The cubic content of the soil which is cultivated
is $\left(\frac{160}{1} \times \frac{121}{4} \times \frac{9}{1} \times \frac{144}{1} \times \frac{8}{1} \times \frac{1}{1728}\right)$ cu. ft.

18. Width of one side of roof

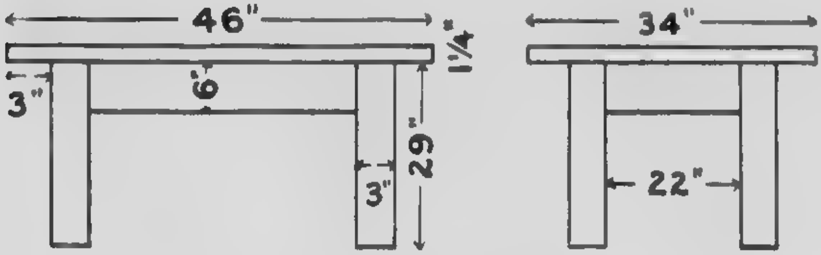
$= (\sqrt{9^2 + 15^2} + 1)$ ft. $= 18\frac{1}{2}$ ft. practically.

Length of roof is 52 ft.

25. The passengers carried in 1911 numbered 37,097,718.
Hence passengers carried in 1913 would be
$\frac{41124181}{1} \times \frac{41124181}{37097718}$.

30. 2 uprights 10" by 67" = 1340 sq. in.
18 partitions 10" by 10" = 1800 " "
7 horizontals 10" by 120" = 8400 " "
Cost at 6c. per sq. ft. = $4.80.

31.



Pieces.		Sq. in.	
1 top	1¼" × 34" × 46" =	2005	
2 side rails	1" × 6" × 36" =	432	
2 end "	1" × 6" × 24" =	288	in.
4 legs			116.R
	Total	2725 sq. in.	116 in.

Now add 2% for waste and find the cost.
The answer is $2.43. . .

32. The circumference is 55 ft. Area to be covered is (55 × 30) sq. ft. Then add 50 sq. ft. for waste.

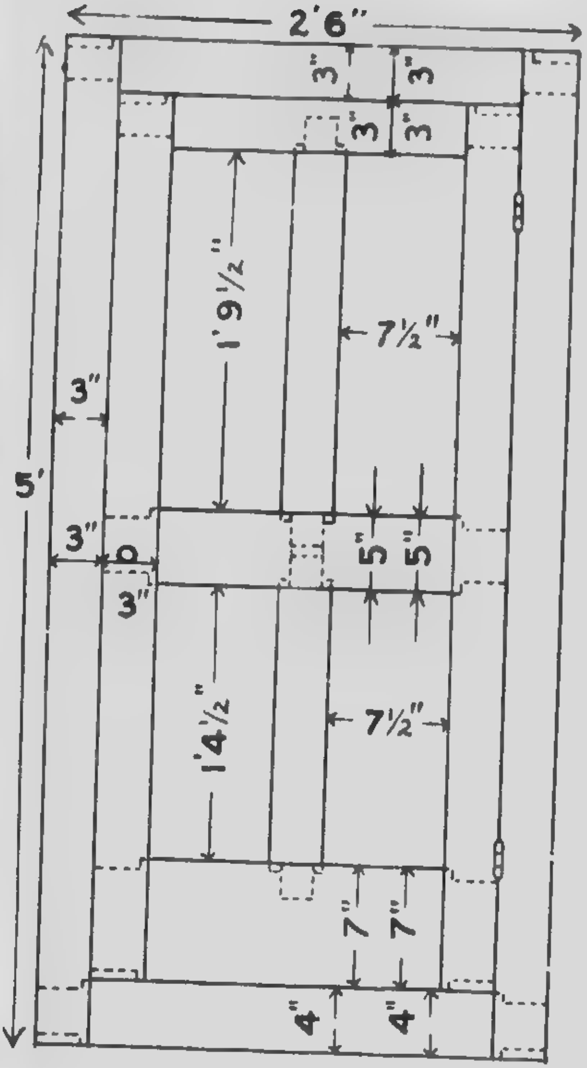
33. Lumber for the sides $= \frac{7 \times 6 \times 4 \times 3}{2}$ board ft.

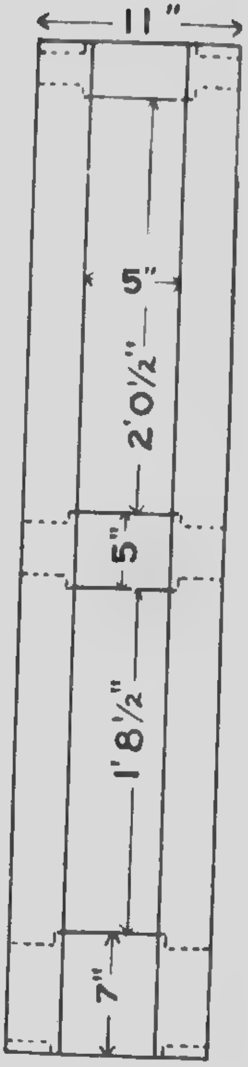
Lumber for the bottom $= \frac{7 \times 7 \times 3}{2}$ board ft.

Lumber for stays $= \frac{2 \times 4}{12} \times \frac{6}{1} \times \frac{7}{1}$ board ft.

" " bottom rests $= \frac{2 \times 4}{12} \times \frac{7}{1} \times \frac{4}{1}$ board ft.

Where fractions occur, figure an extra foot.





			Sq. in.
Front frame.	2 styles @	1" × 3" × 5'0" =	360
	1 upper rail @	1" × 3" × 2'6"	90
	1 lower " @	1" × 4" × 2'6"	120

					Sq. in.
2 end frames,	4 styles	@	1″ × 3″ × 5′0″	=	720
	2 upper styles	@	1″ × 3″ × 11″	=	66
	2 middle "	@	1″ × 5″ × 11″	=	110
	2 lower "	@	1″ × 7″ × 11″	=	154
Shelves,	4	@	1″ × 10½″ × 2′4″	=	1176
Top,	1	@	1″ × 11″ × 2′4″	=	308
Back (in ½″ rebate),	1	@	½″ × 2′5″ × 4′11½″	=	862¾
Door,	2 styles	@	1″ × 3″ × 4′5″	=	318
2 short styles equal to 1 long style	1	@	1″ × 3″ × 4′5″	=	159
	1 upper rail	@	1″ × 3″ × 2′0″	=	72
	1 lock "	@	1″ × 5″ × 2′0″	=	120
	1 lower "	@	1″ × 7″ × 2′0″	=	168
(A plinth and cornice may be added if desired.)					
Panels for ends,	2	@	½″ × 6″ × 2′1½″	=	153
	2	@	½″ × 6″ × 1′9½″	=	129
" for door,	2	@	½″ × 8½″ × 1′10½″	=	191¼
	2	@	½″ × 8½″ × 1′5½″	=	148¾
					5425¾

(5425¾ × $\frac{1}{144}$ × 8) cents = $3.01.

HOME ECONOMICS. PAGE 296

7. Sectional area of the cold-air box = (2⅓ × 1¾) sq. ft.

∴ Area of each hot-air register = ($\frac{1}{7}$ of 2⅓ × 1¾) sq. ft.
= $\frac{7}{12}$ sq. ft.

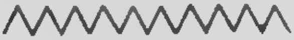
12. 3 acres = 480 sq. rods.

Suppose the field 16 rods by 30 rods and the rows run lengthwise, there will be 85 rows.

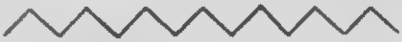
15. A cubic foot of water weighs 1000 oz.

18. 1 lb. chocolate is equivalent to 2 lb. cocoa.

24. The overcasting stitch is a slanting stitch to keep raw edges of seams from ravelling, thus :



⅛ in. wide ; ⅛ in. down.



¼ in. wide ; ⅛ in. down

25. In uneven-basting stitches the stitches and spaces are uneven. The stitches taken upon the needle are about $\frac{1}{3}$ shorter than the space covered by the thread, *e.g.*,

29. The size of the handkerchief is $16'' - (2 \times \frac{1}{4}'') = 15\frac{1}{2}''$.
The distance round the handkerchief is $4 \times 15\frac{1}{2}'' = 62$ in.
Lace for each is $62'' + 4''$ or 66 in.

Lace for all is $\frac{72 \times 66''}{36}$ or 132 yds.

30. Number of yards of cloth $= \frac{36 \times 16''}{36} = 16$ yd.
Cost of goods $= \$12.00$
Cost of lace $= \$26.40$
Cost of spools $= .30$

Number of yards of stitching, handkerchiefs $= \frac{72 \times (62'' - 2'')}{36}$
$= 120$ yds.

" yards of stitching, lace $= \frac{72 \times 62}{36}$
$= 124$ yds.

Cost of stitching $= \$4.88$.
Total cost $= \$43.58$.

32. 3 pints fruit require 2 quarts berries.
(3×12) pints fruit require (2×12) qts. berries or 1 crate.
24 qts. berries require 16 lb. sugar.

34. No. of sheets $= (3 \times 2) \times 2 = 12$ sheets.
Material for sheets $= (2\frac{1}{2} \times 36)'' + 3'' = 93''$
" " 12 sheets $= 31$ yds.
Total cost of sheets $= 31 \times 30$c. $+ 6 \times 8$c.
$= \$9.78$.
No. of pillow cases $= 12$
Material for 1 case $= 36'' + 3'' = 39''$.
" " 12 cases $= 13$ yds.
Total cost of cases $= 13 \times 24$c. $+ 6 \times 6$c.
$= \$3.48$.

35. Length of apron is $9\frac{1}{2}'' + 1\frac{1}{4}'' + \frac{1}{2}''$ or $11\frac{1}{4}''$.
The width of the goods cuts 3 aprons and 3 bands
The width of apron is $13\frac{1}{2}'' + 1''$ or $14\frac{1}{2}''$.
$\therefore$ $21\frac{1}{2}$ in of goods cuts the width of apron and width of strings.
Material for 3 aprons = $21\frac{1}{2}''$ of goods;
$\therefore$ " " 72 aprons = $24 \times 21\frac{1}{2}''$ of goods;
allowing 5% for waste, the material
required is $\frac{105}{100}$ of $\dfrac{24 \times 21\frac{1}{2}}{36}$ yds or $15\frac{1}{20}$ yd

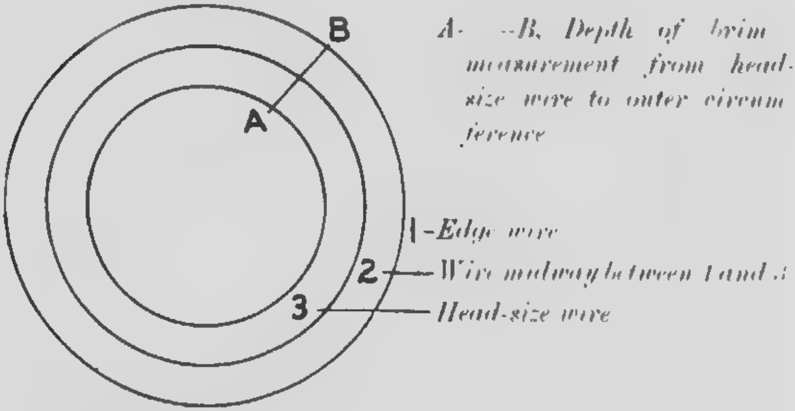
The following millinery problems are given as types of the measurements most commonly required in making hats:

1. What is the radius for each of the following head-size wires (*a*) 23″, (*b*) 25″, (*c*) 26″, (*d*) $23\frac{1}{2}''$, (*e*) $25\frac{1}{2}''$, (*f*) $26\frac{1}{2}''$?

2. Find in each case what measurement on a tape line or ruler is nearest to your answer in the preceding problem

3. How much wire is required on a hat-brim of 22″ head size and 3″ depth of brim, allowing 2″ for lap on each of the circles?

Diagram of hat-brim.



A—B. Depth of brim measurement from head-size wire to outer circumference

1. (*a*) Circumference = 23″.
Radius $= \frac{1}{2}$ of $\frac{7}{22}$ of $23'' = 3\frac{29}{44}$ in.

2. (*a*) A tape line is divided into inches, half-inches, quarter-inches, eighth-inches

[illegible] = 3 [illegible] in. or $3\frac{5}{8}$ in approximately

(*b*) 3 [illegible] in. = 3 [illegible] in or 4 in approximately.

3. (*a*) Head-size, 22

Allowance, 2 .

Radius, $\frac{1}{2}$ of $\frac{7}{22}$ of 24 in = $3\frac{9}{11}$ in

Circumference of edge wire = $\frac{22}{7}$ of 2 ft $6\frac{6}{11}$ in = 43 in nearly

4. Add by $\frac{3}{4}$ inches, as $\frac{3}{4}$″, $1\frac{1}{2}$″, $2\frac{1}{4}$″, 3 etc

" " $\frac{7}{8}$ inches, as $\frac{7}{8}$″, $1\frac{3}{4}$″, $2\frac{5}{8}$″, $3\frac{1}{2}$″, etc

5. In dividing the following head-size wires into eighths, where on the tape line do the divisions come?

(*a*) 21″, (*b*) 22″, (*c*) 25″ (*d*) 27″, (*e*) $24\frac{1}{2}$″. (*f*) $26\frac{1}{2}$

6. (1) Find the radius of each of the following edge wires:

(*a*) 32″, (*b*) 35″ (*c*) 41″, (*d*) $42\frac{1}{2}$″. (*e*) 49

(2) Find in each case what measurement on a tape line or ruler is nearest to your answer

Exercise LXXX Page 308

3. 120 m of silk at 5.45 francs cost 654 francs, or £$\frac{3270}{127}$

$$120 \text{ m} = \frac{120}{.914383} \text{ yd}$$

72 yd. at 6*s*. $7\frac{3}{4}$*d*. sell for £23 18*s*. 6*d*.

$\frac{54.164424}{.914383}$ yd. at 6*s*. 5*d*. sell for £19 0*s*. 1*d*

Total selling price is £42 18*s*. 7*d*. or £$42\frac{223}{240}$

Gain is £$42\frac{223}{240}$ − £$\frac{3270}{127}$ = £$\frac{523681}{30480}$

9. Let cost price be 100%.

Then, if article had cost 20% less, it would have cost 80%.

Then $\frac{\text{gain} + 20}{80} = \frac{\text{gain} + 30}{100}$, since he would gain 20 more if it had cost only 80

15. Proportion is $\frac{1}{3}$: $\frac{2}{7}$: $\frac{3}{8}$, or $\frac{56}{168}$: $\frac{48}{168}$: $\frac{63}{168}$, or 56 : 48 : 63.

First gets $\frac{56}{167}$ of $1686 70.

16. Distance travelled in 1 hr. $= 36 \times 5280$ ft.

Number of revolutions of wheel $= 60 \times 144$.

$$\text{Circumference of wheel} = \frac{36 \times 5280}{60 \times 144}\text{ft.}$$

17. Cost price $= \$1496.50$.

$$\text{Selling price} = \$\frac{1496.50 \times 9}{8}.$$

A note at 90 da for this amount is discounted at 7%.

Discount on $\$100 = \$(\frac{100}{1} \times \frac{7}{100} \times \frac{93}{365}) = \frac{651}{365}$;

$$\therefore \text{proceeds} = \$\left(\frac{1496.50 \times 9}{8} \times \frac{35849}{36500}\right)$$

19 Invoice cost $= 90 \times 2\frac{1}{2}$ fr $= 225$ fr $= \$\frac{225}{5\frac{1}{5}} = \$\frac{1125}{26}$

Total cost, including duty $= \frac{11}{10}$ of $\$\frac{1125}{26} = \$47.596$.

90 metres $= \frac{90 \times 39.4}{36}$ yd. $= 98.5$ yd

Selling price $= 98.5 \times \$1.05 = \$103.425$.

Gain $= \$55.829$

22. Sides are as 4:3; ∴ whatever multiple of 4 the greater side is, the shorter side is the same multiple of 3;

∴ the rectangle contains 12 equal squares

12 squares = 34992 sq. ft.

1 square = 2916 " "

Side of square = 54 ft.; sides of rect = 216 ft and 162 ft

23 Amount $= \$\{2400\,(1.05)^3\}$.

24 80% of marked price = 110% of cost price

26 Surface of sphere $= 4\pi r^2 = 1711\frac{1}{9}$ sq. in., whence $r = \frac{35}{3}$ in

Surface of cube = area of 6 faces = 1734 sq. in,

whence side of cube = 17 in

28. Selling price $= \frac{6}{5}$ cost price $= \$600$; cost price $= \$500$.

" " $= \frac{4}{5}$ " " $= \$600$; " " $= \$750$

Loss on these two $= \$50$; ∴ total gain on third $= \$125$;

∴ third cost $625.

29. Half the sum of the sides $= 60$.

$$\text{Area} = \sqrt{60 \times 35 \times 21 \times 4}.$$

30. Insurance cost $\$\frac{3600}{1} \times \frac{3}{4} \times \frac{8}{1000}$ or \$21.60.
Taxes are $\$\frac{4500}{1} \times \frac{2}{3} \times \frac{18}{1000}$ or \$54.00;
$\therefore$ rent per annum $= \$21.60 + \$40 + \$54 + \$225 = \$340.60$.

31. A draft on Chicago for $\$100\frac{3}{5}$ would sell for \$100 in Denver.

33. Face of note $= \$\frac{730}{1} \times \frac{5}{100} \times \frac{93}{365} + \$730$.
Note is due Sept. 9th. Discounted July 2nd.
Discount $= \frac{73930}{100} \times \$\frac{6}{100} \times \frac{69}{365}$.

35. 10 ac. $= 1600$ sq. rd.; $\therefore$ side of square field $= 40$ rd.
$\pi r^2 = 1600$ sq. rd., whence r may be found.
Circumference $= 2\pi r$.

36. Diagonal of square will be diameter of circle, and the two diagonals intersect at right angles.

37. To cross the bridge the train must travel the length of the bridge + its own length.

39.
$$20 : 10 :: 4^3 : d^3;$$
$$\therefore d = 2\sqrt[3]{4}$$
$$= 3.1748\ldots$$

40. 63^3 cu. in. $= 16^3$ cu. dcm. $= 4096$ l.

41. The grocer cheats $1\frac{9}{17}$ oz. in 16 oz.;

$$\therefore \text{he cheats } \frac{1\frac{9}{17}}{16} \text{ of } \$73.92.$$

42. Area of rectangle of which the base and perpendicular are adjacent sides is (30×40) sq. ft.

The area of rectangle of which the hypothenuse and perpendicular upon it are the dimensions is the same:

$$\text{hence, measure of perpendicular} = \frac{30 \times 40}{50} = 24;$$

$$\therefore \text{perpendicular} = 24 \text{ ft.}$$

45. 6 premiums at $1\frac{1}{4}\% = 7\frac{1}{2}\%$.

$7\frac{1}{2}\%$ of $\frac{4}{5}$ of value of house $= \frac{3}{50}$ of value;

$\therefore$ total loss $= (\frac{1}{5} + \frac{3}{50})$ of value of house $= \$1945$.

46. Area = area of surface of cylinder + twice area of surface of hemisphere

$$= \left(\tfrac{44}{7} \times \tfrac{5}{4} \times \tfrac{6}{1} + \frac{4 \times 22}{7} \times \tfrac{25}{16}\right) \text{ sq. ft.}$$

49. 1 in. $= 2.539954$ cm.

1 cu in. $= (2.539954)^3$ cu. cm. and weighs $\frac{1000}{1728}$ oz.;

$$\therefore\ 1 \text{ cu. cm. weighs } \left(\tfrac{1000}{1728} \times \frac{1}{(2.539954)^3}\right) \text{ oz.}$$

$$= \left(\tfrac{1000}{1728} \times \frac{1}{(2.539954)^3} \times \tfrac{28}{1}\right) \text{ g.}$$

52. Volume of water to raise level in reservoir 2 in.

$$= (156 \times 120 \times \tfrac{2}{36}) \text{ cu. yd.}$$

Area of cross section of pipe $= (\frac{22}{7} \times \frac{7}{1} \times \frac{7}{1})$ sq. in.

$$= \frac{154}{144 \times 9} \text{ sq. yd.;}$$

$$\therefore \text{ length of pipe filled with water} = \left(156 \times 120 \times \tfrac{2}{36} \div \frac{154}{144 \times 9}\right) \text{ yd;}$$

$$\therefore \text{ time required} = \left(\frac{156 \times 120 \times 2}{36} \times \frac{144 \times 9}{154} \times \frac{1}{8 \times 1760}\right) \text{ hr.}$$

53. (25×120) sq. ft. sell for $\$(15 \times 25)$.

54. Rate of train per hour $= \frac{10800}{25} \times \frac{60}{1})$ m.

$$= \left(\tfrac{10800}{25} \times \tfrac{60}{1} \times \frac{1}{.914383}\right) \text{ yd.}$$

59. Volume $= (20 \times 20 \times 4)$ cu. dm.;

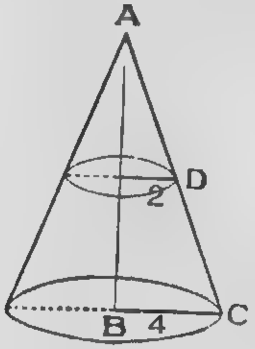
$\therefore$ vessel will contain 1600 l. of water.

60. Selling price $= 87\frac{1}{2}\%$ of $\$4720$.

Amount realised $= \frac{85}{100}$ of $\frac{7}{8}$ of $\$4720$,

and cost price $= \$4720$.

61. Since the straight line which joins the middle points of two sides of a triangle is parallel to the third side and equal to half of it;



$\therefore$ the radius of the upper end of the frustum will be 2 in.

Volume $= \{(\frac{22}{7} \times 2^2 + \frac{22}{7} \times 4^2 + \sqrt{\frac{22}{7} \times 2^2 \times \frac{22}{7} \times 4^2}) \times \frac{5}{1} \times \frac{1}{3}\}$ cu. in.

Since $AB = 10$ and $BC = 4$,

$\therefore AC = \sqrt{116}$; $\therefore CD = \frac{1}{2}\sqrt{116}$.

Then area $= (\frac{22}{7} \times \frac{4}{1} + \frac{22}{7} \times \frac{8}{1}) \times \frac{1}{2} \times \frac{1}{2}\sqrt{116} + \frac{22}{7} \times 2^2 + \frac{22}{7} \times 4^2)$ sq. in.

62. Length of shadow : length of pole :: 4 : 3;

$\therefore$ a pole $48\frac{1}{2}$ ft. long casts a shadow $(48\frac{1}{2} \times \frac{4}{3})$ ft. long.

63. Area of iron in the end of water pipe $= \frac{22}{7}\ \{(7\frac{1}{2})^2 \quad (6\frac{1}{2})^2\}$ sq. in. $= (\frac{22}{7} \times 14 \times 1)$ sq. in.

Cubic contents of pipe $= \frac{\frac{22}{7} \times 14 \times 1 \times 12}{144}$ c. ft. $\quad \frac{11}{3}$ cu. ft.

65. The square on the diagonal = twice area of the courtyard.

67. One side of the square field $= \sqrt{77841}$ yd.;

$\therefore$ length of walk $= (279 \times 4 \times 4 \times 4)$ yd.;

68. The first 7 questions receive 25 marks each;

$\therefore$ 75 marks are to be divided among the last three;

$\therefore$ question number 8 receives $\frac{3}{7+5+3}$ (or $\frac{3}{15}$) of 75 marks.

72. Interest on \$350 for $5\frac{1}{2}$ yr. = \$115.50.

" " \$100 " 1 " $= \$\left(\frac{115.50}{1} \times \frac{2}{11} \times \frac{100}{350}\right) = \$6$;

$\therefore$ rate is 6%.

73. 10000 lb. pork at $8\frac{1}{2}$c. = \$850.

Commission + freight = $3\frac{1}{2}\%$ of \$850 + \$19.55 = \$49.30;

$\therefore$ amount of lumber purchased $= (800.70 \times \frac{100}{102} \times \frac{1}{20})$ thousand ft.

76. See table of linear measurement, page 70.

77. Volume of ice $= (\frac{4840}{2} \times \frac{9}{1} \times \frac{3}{12})$ cu. ft.;
$\therefore$ volume of water $= (\frac{4840}{2} \times \frac{9}{1} \times \frac{3}{12} \times \frac{10}{11})$ cu. ft.

80. Capital at end of first year $=$ 140% of \$4000.
" " " " second " $= 85\frac{5}{7}\%$ of \$5600.
" " " " third " $= 133\frac{1}{3}\%$ of \$4800;
$\therefore$ average annual gain $= \$\frac{2400}{3}$, or \$800.

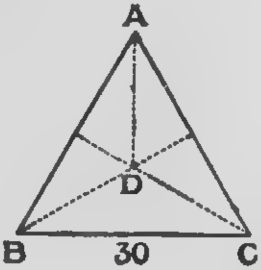
81. There are 13500 oz. of mercury in 1 cu. ft.
" " 2000 × 16 oz. " " $\frac{2000 \times 16}{13500}$ cu. ft.;

$$\therefore \text{number of gallons} = \frac{2000 \times 16}{13500} \times \frac{1728}{1} \times \frac{1}{277.274}.$$

82. *B* gets 10% on \$3000 and *A* gets a salary of \$1000.
A gets $\frac{1}{2}$ of balance of profits $= \frac{1}{2}$ of \$(7460 − 1300);
$\therefore$ in all *A* gets \$1000 + \$3080;
$\therefore$ he may withdraw \$4080 − \$750 = \$3330.

88. By drawing the figure it will be seen that the field consists of a rectangle and a right-angled triangle.

89. Triangles ADC and BDC may be proven equal (Euc. 1, 4); also triangles ADB and BDC;
$\therefore$ triangle BDC is $\frac{1}{3}$ of the triangle ABC.



92. When number is increased 20% it becomes 120% of number. This amount increased $16\frac{2}{3}\%$ becomes $116\frac{2}{3}\%$ of 120% of numoer $= 280$;
$\therefore$ 140% of number $= 280$.

93. Let dimensions be $4x$, $3x$ and $2x$ ft.
Then $24x^3 = 3000$.
Whence $x = 5$.

94. $$9\% \text{ of capital} = \$22500.$$
$$10\% \text{ '' '' } = \$25000;$$
$$\therefore \text{ incomes are } \tfrac{4}{25} \text{ of } \$25000, \text{ etc.}$$

98. House is insured for $\frac{4}{7}$ of \$665, or \$380.
Owner wishes to recover $\frac{5}{6}$ of premium. $\frac{5}{6}$ of $6\% = 5\%$;
$\therefore$ \$95 of value of house is secured by \$100 of insurance.

99. A invests \$5 for 4 mo. $+ \frac{1}{2}$ of \$5 for 8 mo. $=$ \$40 for 1 mo.
B " \$8 " 4 " $+ \frac{1}{3}$ " \$8 " 8 " $= \$53\frac{1}{3}$ " 1 "

$$\therefore A \text{ gets } \frac{40}{40 + 53\frac{1}{3}} \text{ of } \$4000.$$

100. $$\text{Amount of sales} = \$(6.37\tfrac{1}{2} \times 800).$$
$$\text{Amount of commission} = \$\left(6.37\tfrac{1}{2} \times 800 \times \frac{3+2}{102}\right).$$

101. Premium paid first company $= \frac{1}{300}$ of $\frac{1}{5}$ of $\frac{3}{4}$ of \$10000.
" " second " $= \frac{1}{500}$ of $\frac{1}{3}$ of $\frac{3}{4}$ of \$10000.
" " third " $= \frac{3}{700}$ of $\frac{7}{15}$ of $\frac{3}{4}$ of \$10000.

103. Area of hexagonal base = sum of areas of 6 equilateral triangles.
$$= 6\sqrt{1.2 \times .4 \times .4 \times .4} \text{ sq. m.}$$
$$= 1.66272 \text{ sq. m.};$$
$$\therefore \text{ volume of water required} = 3.32544 \text{ cu. m.} = 3325440 \text{ cu. cm.}$$
$$= 3325440 \text{ grams} = 3325.44 \text{ kg.}$$

105. The area of the walk is the difference in area of two circles whose radii are 90 ft. and 115 ft. respectively.

106. $$\text{Amount of note} = \$\left(\tfrac{219}{1} + \tfrac{219}{1} \times \tfrac{92}{365} \times \tfrac{5}{100}\right) = \$221.76.$$
$$\text{Proceeds} = \$221.76 - \$(221.76 \times .07 \times \tfrac{57}{365}).$$

107. Since 1 sq. in. $= .064813669$ sq. dcm.
$\therefore$ the pressure on .064813669 sq. dcm. $= (7000 \times 14.7)$ grains.
But 15.432 grains = 1 gram ;
$$\therefore (7000 \times 14.7) \text{ grains} = \frac{7000 \times 14.7}{.064813669 \times 15.432} \text{ grams}$$
$$= 1028.78 \text{ grams.}$$

108. 1.65% of taxable income = \$33.

110. See Art. 82, page 228, and also solution to Exercise XLVIII., page 109 of Handbook.

111. The square on the diagonal is twice the area of the given square.

$$\pi r^2 = \frac{12^2}{2}. \quad \text{Find } 2\pi r.$$

112. Amount of debt at end of first year = \$15750 − \$2500
= \$13250, etc.

113. $$\text{Breadth of river} = \sqrt{65^2 - 50^2}.$$

115. $$3 \text{ Kg.} = 3000 \text{ g.} = \frac{3000}{8.85} \text{ cu. cm.}$$

$$\text{Length of wire} = \left(\frac{3000}{8.85} \times \frac{1}{.0314159}\right) \text{ cm.}$$

116. \$97½ of value of the building is covered by \$100 insurance.

$\$\frac{5850 \times 4}{5}$ " " " " " "

$$\$\left(\frac{100}{1} \times \frac{2}{195} \times \frac{5850 \times 4}{5}\right) \text{ insurance.}$$

119. Discount for 63 da. = \$11.34.
" " 365 " = \$65.70.

126. 83⅓% of \$2448 worth of goods sell for 106¼% of \$2448 worth.

$\$100$ " " " " " $\frac{106\frac{1}{4}}{83\frac{1}{3}}$ of \$100.

127. Let x = amount of silver added,
then $\frac{9}{20}(1 + x) = \frac{3}{10} + x$.

128. Interest on \$100 for 73 da. at 7% = \$1.40;
∴ interest on \$98.60 for 73 da. = \$1.40.
" " \$100 " 1 yr. = $\$\left(\frac{1.40}{1} \times \frac{365}{73} \times \frac{100}{98.60}\right)$.

142. Deposit Dec. 31, 1890, amounts to \$$\{350\ (1.04)^4\}$
" " 31, 1891, " " \$$\{350\ (1.04)^3\}$, etc.;
∴ total amount at credit on January 1st, 1895,
$= \$[350\{(1.04)^4 + (1.04)^3 + (1.04)^2 + (1.04) + 1\}]$.

143. Account sales of 1000 sheep received from *A*, October 1st, to be sold on commission.

		Sales.				
Oct.	3	250 head at \$5.25			1312	50
	4	525 " " \$6.00			3150	00
	4	5 " killed by accident.				
	6	220 " at \$5.00			1100	00
		Charges.				
Oct.	1	Freight—1000 head at 17½c.	175	00		
	3	Feed and yard—250 head at 2¼c. for 2 da.	11	25		
	4	Feed and yard—530 head at 2¼c. for 3 da.	35	78		
	6	Feed and yard—220 head at 2¼c. for 5 da.	24	75		
		Commission—10% of \$815.72	81	57		
		Balance remitted	5234	15		
			5562	50	5562	50

146. Volume of water $= (462 \times 8 \times 30)$ cu. in.
Surface area of cistern $= (\frac{22}{7} \times \frac{49}{4})$ sq. ft.;
∴ surface of water is lowered $\left(\frac{462 \times 8 \times 30}{1} \times \frac{7}{22} \times \frac{4}{49} \times \frac{1}{144}\right)$ in.

147. Square on diagonal = twice area of square = 55 ac.
Diagonal $= \sqrt{55 \times 4840}$ yd.

154. Mean breadth $= \frac{2 \text{ ft. } 5 \text{ in.} + 2 \text{ ft. } 7 \text{ in.}}{2} = 2$ ft. 6 in.

Mean thickness $= \frac{1 \text{ ft. } 8 \text{ in.} + 1 \text{ ft. } 6 \text{ in.}}{2} = 1$ ft. 7 in.

Volume $= (\frac{41}{3} \times \frac{5}{2} \times \frac{19}{12})$ cu. ft.

155. Slant height $= \frac{\text{twice the area of curved surface}}{\text{circumference of face}}$

158. 7% per annum is $1\frac{3}{4}$% for 3 mo.

\$$98\frac{1}{4}$ is realized when the amount of the note is \$100.

164. Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{110 \times 42 \times 35 \times 33} = 2310$ sq. ft.;

∴ the two parts are $577\frac{1}{2}$ sq. ft. and $1732\frac{1}{2}$ sq. ft.

165. No. of cubic feet of timber submerged $= \frac{3}{4}$ of $\frac{5}{4} \times \frac{5}{4} \times 18$.

Weight of timber $= (\frac{3}{4}$ of $\frac{5}{4} \times \frac{5}{4} \times 18 \times \frac{125}{2})$ lbs.

166. 32 lb. avoir. $= \frac{32 \times 7000}{24 \times 20}$ oz., Troy.

Value in Canadian money $= \frac{32 \times 7000}{24 \times 20} \times \frac{26\frac{13}{16}}{240} \times 486\frac{2}{3}$ ct.

$= \$253.72\frac{41}{72}$.

169. Since $\frac{3.4}{100}$ of mixture $= 850$ lb.;

∴ the mixture $= \frac{100 \times 850}{3.4}$ lb.;

∴ the mixture $= 25000$ lb.,

and the steel $= 24150$ lb.

172. Cu. ft. steel + cu. ft. water weigh $552\frac{1}{2}$ lb.

Cu. ft. steel − cu. ft. water " $427\frac{1}{2}$ lb.;

∴ 2 cu. ft. steel $= 980$

and a cu. ft. steel $= 490$ lb. and a cu. ft. water $= 62\frac{1}{2}$ lb.

177. Size of tank to be filled $= (7 \times 7 \times \frac{22}{7} \times 20 \times 1728)$ cu. in.
Quantity of water flowing per sec. $= (\frac{7}{2} \times \frac{7}{2} \times \frac{22}{7} \times 8 \times 12)$ cu. in.;

$$\therefore \text{ time to fill tank} = \frac{7 \times 7 \times \frac{22}{7} \times 20 \times 1728}{\frac{7}{2} \times \frac{7}{2} \times \frac{22}{7} \times 8 \times 12} \text{ sec.}$$
$$= 24 \text{ min.}$$

183. The areas of the pistons vary as the squares of their like dimensions or as $\frac{100}{9} : 4$;
$\therefore$ lifting force will be $(45 \times \frac{100}{9} \div 4)$ lb. or 125 lb.

185. Weight of beets $= 12\frac{1}{2} \times 400 \times 60$ lb.
" of sugar $= \frac{25}{400}$ of $12\frac{1}{2} \times 400 \times 60$ lb.
$= 18750$ lb.

186. Weight of rails in tons $= \frac{420 \times 1760 \times 80 \times 4}{2000}$
$= 14784$ t.

187. Weight of cream received $= (\frac{63}{400}$ of $2400 + 780)$ lb.
$= 1158$ lb.
Butter manufactured $= \frac{113}{400}$ of 1158 lb.

189. Since 60% of cost of property = \$5000;
$\therefore$ cost of property $= \frac{100}{60}$ of \$5000
$= \$8333\frac{1}{3}$.
Again $\frac{1}{6}$ of cost of property $= \$333\frac{1}{3}$;
$\therefore$ cost of property $= 6 \times \$3333\frac{1}{3} = \$20000$.

193. Area of metal surface of a cross section
$= (\frac{22}{7} \times 9^2 - \frac{22}{7} \times 5^2)$ sq. in.
$= (\frac{22}{7} \times 14 \times 4)$ sq. in.
$\frac{44}{36}$ sq. ft.;
$\therefore$ number of cubic ft. of steel $= \frac{44}{36} \times 12$,
and weight $= (\frac{44}{36} \times 12 \times 490)$ lb.
$= 7186\frac{2}{3}$ lb.

194. $\frac{3}{2}$ of $\frac{7}{8}$ of $\frac{4}{3}$ of cost = \$105;

$\therefore$ cost $= \frac{2}{3}$ of $\frac{8}{7}$ of $\frac{3}{4}$ of \$105

$=$ \$60.

195. No. of revolutions per min. $= \dfrac{36 \times 5280}{60 \times 6\frac{2}{3} \times \frac{22}{7}}$

$= 151\frac{1}{5}$.

196. Area of concrete walk $= (70 \times 105 - 66 \times 99)$ sq. ft.

$=$ 114 sq. yd.

197. Since 1 in. represents $3\frac{1}{4}$ mi.;

$\therefore$ 1 sq. in. " $(3\frac{1}{4} \times 3\frac{1}{4})$ sq. mi.;

$\therefore$ 16 sq. in. " $(16 \times 3\frac{1}{4} \times 3\frac{1}{4})$ sq. mi.

or 108,160 ac.

198. The tax on \$2,450,000 = \$15,925;

$\therefore$ " " " \$5600 $= \$\dfrac{5600 \times 15925}{2{,}450{,}000}$

$=$ \$36.40.

199. Length of lateral side $= \sqrt{(480\frac{3}{4})^2 + 382^2}$

PART III.—SUPPLEMENT

CASTING OUT NINES

1. Any number divided by nine will leave the same remainder as the sum of its digits divided by nine.

This will be evident from the following example:—

$$\begin{aligned}\frac{6783}{9} &= \frac{6000}{9}+\frac{700}{9}+\frac{80}{9}+\frac{3}{9}\\ &= \left(666+\frac{6}{9}\right)+\left(77+\frac{7}{9}\right)+\left(8+\frac{8}{9}\right)+\frac{3}{9}\\ &= \left(666+77+8+\frac{6}{9}+\frac{7}{9}+\frac{8}{9}+\frac{3}{9}\right)\\ &= 751+\frac{6+7+8+3}{9}.\end{aligned}$$

This check will not detect such an error as the interchange of digits, nor any error not affecting the sum of the digits.

2. To check multiplication by casting out the nines.

This test may be given in the form of the following rule:—

Divide the sum of the digits in the Multiplicand by 9, and set down the remainder. Divide the sum of the digits in the Multiplier by 9, and set down the remainder. Multiply the two remainders together, divide the result by 9, and set down the remainder. If the process be correct, this remainder will be the same as the remainder obtained by taking the sum of the digits in the Product and dividing it by 9.

For example, if we multiply 76371 by 854 the product is 65220834.

Sum of digits in Multiplicand = 24,
and 24 ÷ 9 gives remainder 6.

Sum of digits in Multiplier = 17,
and 17 ÷ 9 gives remainder 8

First remainder × second remainder = 48,
and 48 ÷ 9 gives remainder 3.

Sum of digits in the Product = 30,
and 30 ÷ 9 gives remainder 3.

This so-called proof is defective as a proof in the following, as it fails to detect errors in the product—

(*a*) If the order of figures in the product is misplaced, as 37 for 73.

(*b*) If errors be made which counterbalance each other, as 35 written for 62, the sum of the digits in each case being the same.

(*c*) If 9 be written for 0, or 0 for 9, or either be omitted or inserted too often.

ARITHMETICAL COMPLEMENT

3. The arithmetical complement of a number is defined to be the difference between any given number and the unit of the next superior order; thus 6 is the arithmetical complement of 4, 47 of 53, 8468 of 1532, and so on, being the differences respectively of 4, 53, 1532, and 10 100, 10000, the next superior units of these numbers. Conversely, also, 4, 53, 1532 are the arithmetical complements of 6, 47, 8468 respectively.

The arithmetical complement of a number may be found by the following rule:—

Begin at the left hand and subtract every figure from 9 *until the last; subtract that from* 10.

The arithmetical complement may be used to find the difference between two numbers, thus: if 239 be subtracted from 576 the remainder is 337. But if 761, the arithmetical complement of 239, the less number, be added to 576, the greater, the sum will be 1337, one unit (1000 in this case) of the next superior order greater than the difference of the two numbers. By removing this unit, the number will be left equal to the difference of 239 and 576; so that the difference of the two numbers can be found by addition. The arithmetical complement may be written thus $\bar{1}761$, with the subtractive unit on the left, which when added to 576, the sum will be 337, the additive and subtractive units being together equal to zero.

This method is employed with great advantage to find the aggregate of several numbers when some of them are additive and some subtractive. Thus, if we have—

$$3795 - 1532 - 2019 + 8759 - 5104 + 307,$$

we arrange them as follows:—

$$\begin{array}{lr} & 3795 \\ \text{A. C. of 1532 is} & 18468 \\ \text{" \quad 2019 "} & \bar{1}7981 \\ & 8759 \\ \text{" \quad 5104 "} & \bar{1}4896 \\ & 307 \\ \hline & 4206 \end{array}$$

the aggregate required.

4. To Square any Number.

$$\begin{aligned} 78^2 &= 80 \times 76 + 2^2 = 6084 \\ 34^2 &= 30 \times 38 + 4^2 = 1156 \\ 75^2 &= 80 \times 70 + 5^2 = 5625 \\ 113^2 &= 110 \times 116 + 3^2 = 12769 \end{aligned}$$

Add to and subtract from the given number such a number as is necessary to form an exact number of tens. Multiply the resulting number by the number of tens thus found and to the product add the square of the number added and subtracted. The sum is the square of the given number.

5. To multiply using Arithmetical Complements when there are the same number of figures in the Multiplier and the Multiplicand.

Ex. Multiply 987 by 994.

A. C. of multiplicand = 13
A. C. of multiplier = 6
6 × 13 = 78; 078
987 − 6 = 981; 981078 = product required.

Hence, *multiply the A. C. of the multiplicand by the A. C. of the multiplier. Set down the product, filling up as many places to the left with naughts as will make the number of places equal to the number of figures in the multiplicand. Subtract the A. C. of the multiplier from the multiplicand and set the remainder to the left of the figures already placed. The result will be the product required.*

6. Tests of Exact Divisibility.

(*a*) A number is exactly divisible by 2 if its right-hand figure is zero or an even digit.

(*b*) A number is exactly divisible by 4 if its two right-hand figures are zeros or express a number exactly divisible by 4.

(*c*) A number is exactly divisible by 8 if its three right-hand figures are zeros or express a number exactly divisible by 8.

(*d*) A number is exactly divisible by 3 if the sum of its digits is exactly divisible by 3.

(*e*) A number is exactly divisible by 9 if the sum of its digits is exactly divisible by 9.

(*f*) A number is exactly divisible by 5, 25 or 125 if the number ends in 1, 2 or 3 zeros, or if the number expressed by the right-hand figure, or by the two, or by the three right-hand figures is exactly divisible by 5, 25 or 125 as the case may be.

(*g*) A number is exactly divisible by 11 when the difference between the sum of the digits in the odd places and the sum of the digits in the even places is either 0 or exactly divisible by 11. Thus, 24794 and 829191 are exactly divisible by 11.

CUBE ROOT

7. When a number is multiplied by itself twice, the result is called the **Cube** of the number. Thus 27 is the cube of 3, and 216 is the cube of 6.

8. The **Cube Root** of a given number is that number whose cube is equal to the given number.

Thus the Cube Root of 343 is 7, because the cube of 7 is 343.

The symbol $\sqrt[3]{}$, placed before a number, denotes that the cube root of that number is to be taken; thus $\sqrt[3]{125}$ is read "the cube root of 125."

9. A number which has an integer for its cube root is called a **Perfect Cube.**

10. To find the Cube Root of a perfect cube, greater than 1000, we proceed by a rule which we shall now explain.

Ex. To find the Cube Root of 91125.

	4		91,125
			64
12	5	4800	27125
		625	
		5425	27125

First divide the number 91125 into two periods by drawing a line marking off *three* figures on the right.

Then take the nearest perfect cube not greater than 91, which is 64, and set down its cube root, which is 4, in a line with 91125, and some way to the left. This is the first figure of the root.

Then subtract 64 from 91, and to the remainder attach the second period, 125.

Now place three times the first figure of the root, 12, to the extreme left, and three times the square of the first figure of the root, 48, with two zeros annexed to it, just on the left of the 27125.

Divide 27125 by 4800, and set the quotient, 5, midway between 12 and 4800. Then read 12 5 as 125; multiply this by 5; put the result, 625, under the 4800; add to it the 4800; this gives 5425; multiply this by 5; put the result, which is 27125, under the first remainder; subtract, and as there is no remainder, the process is complete, and the root is 45.

Examples I

Find the Cube Roots of

1. 4096.
2. 32768.
3. 74088.
4. 493039.
5. 614125.
6. 262144.
7. 39304.
8. 389017.
9. 778688.
10. 970299.
11. 59319.
12. 250047.

Next, let us take the case in which the cube root has three figures and extract the cube root of 428661064.

	7		428|661|064
			343
21	5	14700	85661
		1075	
		15775	78875
		25	
225	4	1687500	6786064
		9016	
		1696516	6786064

We separate the number 428661064 into three periods, and take the nearest perfect cube not greater than 428, which is 343, and we set down its cube root, which is 7. We then subtract 343 from 428, and annex to the remainder 661, the second period.

Then we set down three times 7, which is 21, and three times the square of 7, which is 147, and annex two zeros to it.

Then we divide 85661 by 14700, which gives the quotient 5, and this we put down midway between 21 and 14700.

Then we multiply 215 by 5, which gives 1075; we add this to 14700; we multiply the result, 15775, by 5; and subtract the product, 78875, from 85661; and to the remainder we annex the third period, 064.

We then set down three times 75, which is 225,
and three times the square of 75, which is 16875.

N.B. This last result can be obtained by setting the square of 5, the second figure of the root, under the second divisor, and adding the three numbers coupled by the bracket.

We then annex two zeros to 16875 and repeat the process explained above to find 4, the third figure of the cube root, which is in this case 754.

Examples II

Find the Cube Roots of

1. 14706125.
2. 149721291.
3. 28934443.
4. 300763000.
5. 2097152.
6. 5735339.
7. 99252847.
8. 1092727.
9. 16777216.
10. 194104539.
11. 84027672.
12. 130323843.
13. 322828856.
14. 354894912.
15. 700227072.
16. 134217728.
17. 122615327232.
18. 673373097125.

11. To extract the Cube Root of a Decimal Fraction.

In order that a Decimal Fraction may be a Perfect Cube, it must be of the 3rd, 6th, 9th . . . . order, the index of the order being some multiple of 3.

We then proceed in the following way :—

Ex. 1. To find the Cube Root of .343.

$$\sqrt[3]{.343} = \sqrt[3]{\frac{343}{1000}} = \frac{7}{10} = .7.$$

Ex. 2. To find the Cube Root of .039304.

$$\sqrt[3]{.039304} = \sqrt[3]{\frac{39304}{1000000}} = \frac{34}{100} = .34.$$

Ex. 3. To find the Cube Root of .012812904.

$$\sqrt[3]{.012812904}=\sqrt[3]{\frac{12812904}{1000000000}}=\frac{234}{1000}=.234.$$

12. To extract the cube root of an integer or decimal expression to a particular place of decimals, in the given expression, we must take *three times the number* of decimal places required.

Thus, to find the cube root of 4.23 accurately to three places of decimals, we extract the cube root of 4.230000000, making the given expression a decimal of the *ninth* order. In working this example, we find the cube root of 4.230000000, *regarded as a whole number,* and mark off three decimal places in the result.

13. The Cube Root of a *Vulgar Fraction* may be found by taking the roots of the numerator and denominator, or by reducing the fraction to a decimal of the 3rd, 6th, 9th . . order, and proceeding as in Art. 12.

Examples III

Find the Cube Root of

1. .389017.
2. .048228544.
3. 27054.036008.
4. $\frac{1331}{1728}$.
5. $\frac{250}{686}$.
6. $5\frac{13}{343}$.
7. $405\frac{28}{125}$.

Find to three places of decimals the Cube Roots of

8. 5.
9. 576.
10. .121861281.
11. 15.926972504.
12. $\frac{5}{9}$.
13. $\frac{3}{4}$.
14. $\frac{1}{5}$.
15. $7\frac{3}{5}$.
16. $3\frac{1}{5}$.

14. The *fourth* root of a number is found by taking the square root of the square root of the number.

Thus $\sqrt[4]{4096}=\sqrt{64}=8$.

The *sixth* root of a number is found by taking the cube root of the square root of the number,

Thus $\sqrt[6]{64}=\sqrt[3]{8}=2$.

Examples IV

Find the Fourth Roots of

1. 531441. 2. 4100625. 3. 1575.2961.

Find the Sixth Roots of

4. 4826809. 5. 24794911296. 6. 282429.536481.

Examples V

1. The product of three equal numbers is 679151439 Find one of them.

2. There are three numbers, the second is twice the first, and the third is twice the second. The product of the three is 5000211000. Find the largest number.

3. Of three numbers the second is $\frac{1}{3}$ of the first and the third is twice the second. Their product is 178746. Find the numbers.

4. There are four numbers, the second being twice the first, the third twice the second, and the fourth twice the third. Their continued product is 585640000. Find the numbers.

5. Write down the squares of 8, 9, 10 and 11, and from these squares derive a rule for finding the square of any number when the square of the number next greater or next less to it is known.

6. The square of 5987 is 35844169. From this find the square of 5988 and of 5986.

7. Resolve the number 3456649728 into prime factors and from these determine the cube root of the given number.

8. Find the ninth root of 387420489.

15. The extraction of the cube root, by the ordinary rule, is a troublesome process, seldom used and easily forgotten. The following process is much simpler and more easily remembered.

Let a be an approximate value of the cube root of N, so that $\sqrt[3]{N} = a + x$, x being very small;

then $$N = (a+x)^3 = a^3 + 3a^2x + 3ax^2 + x^3$$
$$= a^3 + 3ax(a+x), \text{ nearly, since } x \text{ is small.}$$

First suppose $$N = a^3 + 3a^2x;$$
$$\therefore\ x = \frac{N - a^3}{3a^2};$$

and $$\therefore\ a + x = \frac{N + 2a^3}{3a^2};$$

therefore more nearly, $$N = a^3 + 3ax\frac{N + 2a^3}{3a^2}$$

and $$x = \frac{N - a^3}{N + 2a^3}a,$$

and, therefore, $$\sqrt[3]{N} = a + x$$
$$= \frac{2N + a^3}{N + 2a^3}a.$$

Suppose we want to find the cube root of any number N. In the first place we find some number a whose cube is somewhere near the given number. Then the fraction,

$$\frac{2N + a^3}{N + 2a^3}a$$

will be a nearer approximation to the cube root than a itself was. When we have found this value, we can take this as a and repeat the process.

Thus, to find the cube root of 241.804367, we observe that 216, the cube root of 6, is nearest to 241. Hence the first value of a is 6.

Therefore, $$\frac{2N + a^3}{N + 2a^3}a$$
$$= \frac{699.608734}{673.804367} \times 6$$
$$= \frac{4197.652404}{673.804367}$$
$$= 6.23, \text{ very nearly.}$$

On trying 6.23, we find it is correct.

Ex. Find the Cube Root of 47.

The nearest cube to 47 is that of 4.

Hence,

$$\frac{2N+a^3}{N+2a}\cdot a$$

$$=\frac{94+64}{47+128}\times 4$$

$$=\frac{632}{175}$$

$$=\frac{2528}{700}$$

3.61, nearly.

Next, take 3.61 for *a*, and substitute in the formula, and we get 3.6088261, which is correct to seven places of decimals.

Examples VI

Find to four places of decimals the Cube Root of

1. 12.
2. 30.
3. 225.
4. 375.
5. .9.
6. .08.
7. 9.27.
8. .587.
9. 8.76.

COMMISSION

16. Manufacturers and others frequently employ a third person to sell their goods or to buy goods for them. This third party is called a *Commission Merchant* or *Agent*. The pay he receives for his services is called *Commission*.

An *agent* receives a consignment of 400 head of cattle to sell. He is to be allowed 3 per cent. of the *gross proceeds* for making the sale, and to remit the *net proceeds* to the *consignor*. He sells the cattle at \$42 per head.

The gross proceeds = \$42 × 400 = \$16800.
Agent's commission = \$16800 × $\frac{3}{100}$ = \$504.
The net proceeds = \$16800 − \$504 = \$16296.

The commission is $\frac{504}{16296}$, or $\frac{3}{97}$ of the net proceeds, and $\frac{3}{100}$ of the gross proceeds.

Suppose Mr. A. instructs the *agent* to buy for him 600 sheep at \$7.50 each and agrees to allow him 3 per cent. of the cost of the sheep for making the purchase.

$$\text{Total cost of sheep} = \$7.50 \times 600 = \$4500.$$
$$\text{Commission for buying} = \$4500 \times \tfrac{3}{100} = \$135.$$

Therefore Mr. A. must send the agent \$4500 + \$135, or \$4635. The commission is $\frac{135}{4635}$ or $\frac{3}{103}$ of the sum remitted, and $\frac{3}{100}$ of the sum invested.

PRESENT WORTH AND TRUE DISCOUNT

17. Suppose *A* owes *B* \$105, to be paid at the end of a year. If *A* be disposed to pay off the debt at once, the sum which he ought to pay should be such that, if put out at interest by *B*, it will amount at the end of a year to \$105. Suppose, further, that *B* can put out his money at 5 per cent. interest. Then, if he put out \$100 at interest, this is the sum which will amount at the end of a year to \$105.

Hence, \$100 is the sum which *A* ought to pay at once, and this is called the *Present Worth* of the debt, and is evidently such a sum as would, if put out at interest for the given time and rate, amount to the debt. The difference between the Debt and the Present Worth, which is in the case under consideration \$5, is called the Discount.

This kind of Discount is known as *Mathematical* or *True Discount*, and is to be distinguished from both Trade and Bank Discounts.

Its computation falls under the case of Simple Interest in which the Amount, Rate and Time are given to find the Interest or the Principal. In True Discount the Debt corresponds to the Amount, the Present Worth to the Principal and the True Discount to the Interest.

There are, however, a variety of problems which may arise in connection with True Discount, some of which are illustrated below.

Ex. 1. Thus, to find the Present Worth of \$1781.40, due 4 years hence, reckoning interest at 5 per cent.

The interest on \$100 for 4 yr. at $5\% = \$20$;

$\therefore$ \$120 has for its present worth \$100;

$\therefore$ \$1 has for its present worth $\$\frac{100}{120}$;

$\therefore$ \$1781.40 has for its present worth $\$\frac{1781.40 \times 100}{120}$,

or \$1484.50;

$\therefore$ present worth required $= \$1484.50$.

Ex. 2. Find the Discount on \$1781.40, due 4 years hence, reckoning interest at 5 per cent.

The interest on \$100 for 4 yr. at $5\% = \$20$;

$\therefore$ \$120 has for its discount \$20;

$\therefore$ \$1 has for its discount $\$\frac{20}{120}$;

$\therefore$ \$1781.40 has for its discount $\$\frac{1781.40 \times 20}{120}$

$= \$296.90$.

Ex. 3. What was the debt of which the Discount for 8 months at 9 per cent. was \$44.46?

The interest on \$100 for 8 mo. at $9\% = \$6$;

$\therefore$ \$6 is the discount on \$106;

$\therefore$ \$1 is the discount on $\$\frac{106}{6}$;

$\therefore$ \$44.46 is the discount on $\$\frac{44.46 \times 106}{6}$

$= \$785.46$.

Ex. 4. The interest on a certain sum of money for two years is \$50, and the discount for the same time and rate is \$45. Find the sum and the rate per cent. per annum.

Since \$50 is the interest on a sum of money which sum

= (its present worth + its discount)

= (its present worth + \$45)

and \$45 is the interest on its present worth;

$\therefore$ \$5 is the interest on \$45;

$\therefore$ \$1 is the interest on $\$\frac{45}{5}$;

∴ \$50 is the interest on \$$\frac{50 \times 45}{5}$, or \$450;

∴ \$450 is the sum required.

Again, the interest on \$45 for 2 yr. = \$5;

∴ the interest on \$45 for 1 yr. = \$$\frac{5}{2}$;

∴ the interest on \$1 for 1 yr. = \$$\frac{5}{45 \times 2}$;

∴ the interest on \$100 for 1 yr. = \$$\frac{100 \times 5}{45 \times 2}$ = \$$5\frac{5}{9}$;

∴ the rate = $5\frac{5}{9}$%.

NOTE I. *From the above it will be seen that the discount on any sum is the present worth of the interest of that sum for the same time and rate. Thus, \$45 is the present worth of \$50 for 2 yr. at a certain rate per cent.*

Ex. 5. If \$20 be allowed off a bill of \$420 due in 6 months, how much shall be allowed off the same bill due in 12 months?

The discount off \$420 for 6 mo. = \$20;

∴ the interest on \$400 for 6 mo. = \$20;

∴ the interest on \$400 for 12 mo. = \$40;

∴ the discount off \$440 for 12 mo. = \$40;

∴ the discount off \$1 for 12 mo. = \$$\frac{40}{440}$;

∴ the discount off \$420 for 12 mo. = \$$\frac{420 \times 40}{440}$ = \$$38\frac{2}{11}$;

∴ the discount required = \$$38\frac{2}{11}$.

NOTE II.—*The student will observe that the discount is not proportioned to either the time or the rate.*

Ex. 6. If \$15 be the interest on \$115 for a given time, what should be the discount off \$115 for the same time?

The interest on \$115 = \$15;

∴ the discount off \$130 = \$15;

∴ the discount off \$1 = \$$\frac{15}{130}$;

∴ the discount off \$115 = \$$\frac{115 \times 15}{130}$ = \$$13\frac{7}{26}$;

∴ the discount required = \$$13\frac{7}{26}$.

Ex. 7. If \$10 be allowed off a bill of \$110, due 8 months hence, what should be the bill from which the same sum is allowed as 4 months' discount?

\$10 is the discount off \$110 for 8 mo.;
∴ \$10 is the interest on \$100 for 8 mo.;
∴ \$10 is the interest on \$200 for 4 mo.;
∴ \$10 is the discount off \$210 for 4 mo.;
∴ the sum required = \$210.

Examples VII

1. Find the present worth of
   (*a*) \$5520, due 4 yr. hence, at 5%.
   (*b*) \$84.70, due $2\frac{1}{3}$ yr. hence, at 9%.
   (*c*) \$615, due 1 yr. 4 mo. hence, at 7%.
   (*d*) \$1120, due 16 mo. hence, at 5%.
   (*e*) £618 2*s*. 6*d*., due $3\frac{3}{4}$ yr. hence, at 4%.

2. Find the discount on
   (*a*) \$636, due in 9 mo. at 8%.
   (*b*) \$1884.30, due in $3\frac{1}{8}$ yr. at 10%.
   (*c*) \$637.50, due in $5\frac{1}{2}$ yr. at 5%.
   (*d*) £1165 16*s*. 3*d*., due in $2\frac{1}{2}$ yr. at 6%.
   (*e*) £252 19*s*. 3*d*., due in 9 mo. at $4\frac{1}{3}$%.

3. Find the present worth of \$6934.50, due 3 yr. hence, at 5%.

4. Find the discount on \$68.40, due $1\frac{1}{4}$ yr. hence, at $5\frac{1}{2}$%.

5. A tradesman accepts \$19.3125 in payment of a debt of $\$20\frac{587}{960}$, due in 12 mo., in consideration of being paid at once. What rate of discount does he allow?

6. Find the present worth of a bill for \$1127.10, drawn Jan. 1 at 4 mo., and discounted Feb. 20 at 10% per annum.

7. The discount on \$275 for a certain time is \$25. What is the discount on the same sum (*a*) for twice that time, and (*b*) for half that time?

8. A tradesman marks his goods with two prices, one for cash and the other for credit of 6 mo. What relation should the prices bear to each other, allowing interest at $7\frac{1}{2}\%$? If the credit price of an article be \$33.20, what is the cash price?

9. If \$98 be accepted in present payment of \$128, due some time hence, what should be the proper discount of a bill of \$128 which has only half the time to run?

10. A certain sum ought to have \$20.80 allowed as 8 mo. interest on it. But a bill for the same sum due in 8 mo. at the same rate should have \$20 only allowed off as discount in consideration of present payment. What is the sum and the rate per cent.?

EQUATION OF PAYMENTS

18. When several sums of money are due from *A* to *B*, payable at different times, it is often required to find the time, called the **Equated Time,** at which all may be paid together, without injustice to *A* or *B*.

When great exactness is demanded, interest must be added to the sums paid after they are due, and discount subtracted from the sums paid before they are due. But in practice the following Rule is sufficiently accurate :—

Multiply each debt by the number of days (or months) after which it is due. Add the results together. Divide this sum by the sum of the debts. The quotient will be the number of days (or months) in the equated time.

Take the following examples :—

Ex. 1. If \$300 be due from *A* to *B* at the end of 5 months, and \$700 at the end of 9 months, when may both sums be paid in a single payment without unfairness to *A* or to *B*?

$$\text{Number of months in equated time} = \frac{300 \times 5 + 700 \times 9}{300 + 700} = \frac{7800}{1000} = 7\tfrac{4}{5};$$

∴ the whole amount of the debt should be paid at the end of $7\frac{4}{5}$ mo.

The principle on which this solution depends is that the interest of the money, the payment of which is delayed beyond the time at which it is due, is equal to the interest of that which is to be paid before it becomes due.

In the above example \$300 is kept $2\frac{4}{5}$ months after it is due, and the interest on it for that time is the same as the interest on \$840, \$$(300 \times 2\frac{4}{5})$ for one month.

But \$700 is paid $1\frac{1}{5}$ months before it is due, and the interest on it for that time is the same as the interest on \$840, \$$(700 \times 1\frac{1}{5})$ for one month.

Ex. 2. *A* is indebted to *B* in the following amounts: \$500, due in 6 months; \$600, due in 7 months; \$800, due in 10 months. Find the time when all these payments should be made together.

$$\begin{array}{rr}
500 \times 6 = & 3000 \\
600 \times 7 = & 4200 \\
800 \times 10 = & 8000 \\
\hline
1900 \quad 1900) & 15200 \\
\hline
 & 8;
\end{array}$$

$\therefore$ the equated time $= 8$ mo.

Note.—*This method is but a rough approximation, and can only be taken as equitable when the various times of payment are not widely apart. It will, in short, be applicable only to cases which occur in the ordinary course of trade, and is therefore all that we require in the present work.*

It is also to be observed that the error involved in this method is *slightly in favor of the payer*, because interest is calculated on the payments made before they are due, instead of discount, in the algebraical process from which the method is derived.

Examples VIII

1. What is the equated time of \$250 due 4 mo. hence, and \$350 due 10 mo. hence?

2. Find the equated time of $300 due in 3 mo. hence, $400 due in 4 mo. hence, and $500 due in 6 mo. hence.

3 On Jan. 15, I bought a bill of goods amounting to $900, $275 of which was on 30 da. credit, $300 on 60 da., and $325 on 90 da. On what date should the debt be discharged by one payment ?

4. A debt of $2400 was contracted on March 6. 1896, payable in 8 mo., but $400 was paid in 2 mo., $600 in 5 mo., $800 in 7 mo. What was the equitable time for paying the balance ?

5. The sum of $1200 is due in 14 mo. If $\frac{1}{3}$ of the sum be paid in 9 mo., and $\frac{1}{5}$ of it in 13 mo., in what time ought the remainder to be paid ?

6. One half of a debt of $1000 is due in 10 mo., $\frac{1}{4}$ of it in 12 mo., $\frac{1}{8}$ in 16 mo. and the remainder in 20 mo. When might the whole be paid at one payment ?

7. A debt is due in 12 mo. hence, but $\frac{1}{6}$ of it is paid in 6 mo., and $\frac{1}{18}$ in 9 mo. When should the remainder be paid ?

8. Of a debt of $1400, $100 is due immediately, $600 at the end of 1 mo., $400 at the end of 7 mo., and the remainder at the end of a year. At what time might the whole debt fairly be paid in one sum ?

9. A grocer ought to receive from a customer $50 at the end of 2 mo., $30 at the end of 4 mo., and $20 at the end of 6½ mo. What would be the proper time for receiving the whole sum together ?

10. A debt is to be paid as follows: One-sixth now, and one-sixth every 3 mo., until the whole is paid. When might the whole debt be paid at once ?

EQUATION OF ACCOUNTS

19. Equation of Accounts (also called "Averaging of Accounts" and "Compound Equation of Payments") is the process of finding at what time the *balance of an account* can be paid without gain or loss to either party.

The **Balance of an Account** is the difference between the two sides of it, and is what one owes the other.

Ex. T. Black's Account in our Ledger.

Dr. T. Black. Cr.

1900.			1900.		
Jan. 1	To Mdse.	$500.00	Feb. 10	By Cash	$1000.00
Feb. 4	" "	600.00	Mar. 4	" "	600.00
Mar. 10	" "	800.00			

Take the latest date, Mar. 10, as focal date.

Due Dates.	Days.		When Paid.	Days.	
Jan. 1	500 × 68 =	34000	Feb. 10	1000 × 28 =	28000
Feb. 4	600 × 34 =	20400	Mar. 4	600 × 6 =	3600
Mar. 10	800 × 0 =	0			
	1900	54400		1600	31600
	1600	31600			
Bal. of ac.	300	22800	Bal. of products.		

Number of days = 22800 ÷ 300 = 76.
Count 76 days backward from Mar. 10;
∴ $300 is due Dec. 24, 1899.

The debit side shows the amount Black has received.

The credit side shows the amount Black has given.

Suppose the account to be settled on the latest date, March 10.

Dr.

Then, interest on Interest on

$500 for 68 da. = $1 for 500 × 68 da., or 24000 da.
600 " 34 " = 1 " 600 × 34 " " 20400 "
800 " 0 " = 1 " 500 × 0 " " 0 "
"
$1900 = 1 for 54400 "

Cr.

Interest on   Interest on
$1000 for 28 da. = $1 for 1000 × 28 da., or 28000 da.
600 " 6 " = 1 " 600 × 6 " " 3600 "
$1600 = $1 for 31600 "
∴ Black receives $1900 and the int. on $1 for 54400 da.
" gives $1600 " " " " $1 " 31600 "
" owes $300 " " " " $1 " 22800 "
or " " $300 " " " " $300 " $\frac{22800}{300}$ " or 76 da.;
∴ $300 is due 76 days before March 10, or Dec. 24.

(a) *Find the date when each item is due or paid on both sides.*

(b) *Take the latest due date on either side, thus found, for the focal date. Multiply each item on both sides of the account by the number of days between the focal date and the date of the item.*

(c) *Add the products on each side, and subtract the sum of those on the one side from the sum of those on the other, and divide the difference by the balance of the account. The quotient is the number of days to be counted forward from the focal date, when the balance of the products and the balance of the account are on opposite sides, and backward from the focal date when they come on the same side.*

Examples IX

1. In the following account it is required to find the balance, and when it is due.

Dr. James Adamson. Cr.

1900.		$	1900.		$
Mar. 9	To merchandise . .	300	Mar. 20	By cash	247
May 12	" " . .	474	April 11	" draft at 30 da . .	400
June 19	" " . .	564	July 10	" cash	260

2. Find the equitable balance of the following account :—

Dr. JOHN JONES IN ACCOUNT WITH WM. SMITH. Cr.

1900.		$	1900.		$
Oct. 2	To merchandise....	180	Nov. 18	By mdse., 2 mo...	150
Nov. 8	" " 3 mo.	120	Dec. 24	" cash..........	200
Dec. 16	" " 4 mo.	240			

3. Find the equated time for paying the balance of the following account :—

Dr. SCOTT, HUGHES & Co. Cr.

1899.			1899.		
Jan. 29	Mdse., 2 m.	$519.00	Jan. 2	Mdse., 3 m.	$357.00
Feb. 5	Mdse., 3 m.	423.00	" 25	Mdse., 2 m.	738.00
" 19	Mdse., 2 m.	969.00	Feb. 4	Mdse., 3 m.	421.00

ALLIGATION

20. Alligation is the process by which we find the mean or average price of a compound when we mix or unite two or more articles of different values.

Ex. 1. A merchant has brown sugar worth 8 cents per pound, New Orleans worth 9 cents, and refined sugar worth 14 cents. How many pounds of each kind must he use in order to form a mixture worth 12 cents per pound?

By selling the mixture at 12c. per pound we see that 8c. (brown) gains 4c. on 1 lb.

∴ 1c. is gained on $\frac{1}{4}$ lb.

9c. (New Orleans) gains 3c. on 1 lb.;

∴ 1c. is gained on $\frac{1}{3}$ lb.

14c. (refined) loses 2c. on 1 lb.;

∴ 1c. is lost on $\frac{1}{2}$ lb.

Now with every cent *gain* he must combine a cent *loss*; hence he must have

$$\left.\begin{array}{l} \frac{1}{4}\text{ lb. at } \ 8\text{c.} \\ \frac{1}{2}\text{ lb. at } 14\text{c.} \\ \frac{1}{3}\text{ lb. at } \ 9\text{c.} \\ \frac{1}{2}\text{ lb. at } 14\text{c.} \end{array}\right\} = \left\{\begin{array}{l} 3\text{ lb. at } \ 8\text{c.} \\ 6\text{ lb. at } 14\text{c.} \\ 4\text{ lb. at } \ 9\text{c.} \\ 6\text{ lb. at } 14\text{c.} \end{array}\right.$$

He must, therefore, have 3 lb. of brown sugar, 4 lb. New Orleans, and 12 lb. refined.

We may show that these quantities will make the mixture required, as follows:

3 lb. at 8c. per lb. = 24c.
4 lb. at 9c. " = 36c.
12 lb. at 14c. " = 168c.
19 lb. = whole mixture. 228c. = value of mixture.
Hence, if 19 lb. be worth 228c.
1 lb. is worth $\frac{228}{19} = 12$c.

Or we may reason thus: The 1c. *gained* on the ¼ lb. of brown exactly balances the 1c. *lost* on the ½ lb. of the refined. Hence he must take ¼ lb. of the brown and ½ lb. of the refined, or 2 lb. of one and 4 lb. of the other.

Similarly, for every 2 lb. of New Orleans there must be 3 lb. of refined. As 4 lb. of refined were required to balance the brown, and 3 lb. of the refined to balance the New Orleans, there must be 7 lb. of the refined in the compound. Therefore, the respective quantities are 2 lb. brown, 2 lb. New Orleans, and 7 lb. refined.

From the above we see that in examples of this kind a variety of answers may frequently be obtained, and all of them may be correct. To ascertain their correctness we resort to the method of proof given in this example.

From the above analysis we derive an easy practical method of solving such questions.

Ex. 2. How much sugar at 10, 13, 15, 17 and 18c. per pound must be taken to make a mixture worth 16c. per pound?

We proceed as follows :—

Differences.	16	
6	10	1
3	13	1
1	15	1
..	.	..
1	17	2, 4, 6, 8
2	18	4, 3, 2, 1

Write down the prices in a vertical column, and place the *differences* between these prices and the *mean* in a second vertical column to the left. Now take 1 at 10, 1 at 13, and 1 at 15 (the lowest that could be taken); this would represent a *loss* of 10 as compared with the *mean;* and this loss must be balanced by taking the necessary multiples of the differences, 1 and 2, which represent *gain*, as compared with the mean.

It is seen that this loss of 10 can be made up in *four* ways. By 2 at 17, 4 at 18; 4 at 17, 3 at 18; 6 at 17, 2 at 18; 8 at 17, and 1 at 18.

Other combinations may be made, as *e.g.*,

6	10	1
3	13	1
1	15	2
..	..	..
1	17	1, 3, 5, 7, 9
2	18	5, 4, 3, 2, 1

Here 1 at 10, 1 at 13, and 2 at 15, give *loss* of 11, which can be made up by multiples of the differences 1 and 2 (opposite 17 and 18) in *five* ways, as indicated.

Also,

6	10	1
3	13	2
1	15	1
.	..	..
1	17	1, 3, 5, 7, 9, 11
2	18	6, 5, 4, 3, 2, 1

When 1 at 10, 2 at 13, and 1 at 15 give 13 loss, which may be made up in *six* different ways.

Again,

6	10	2
3	13	1
1	15	1
..	..	..
1	17	2, 4, 6, 8, 10, 12, 14
2	18	7, 6, 5, 4, 3, 2, 1

Where 2 at 10, 1 at 13, and 1 at 15 give *loss* of 16, which may be made up in *seven* ways.

Also,

6	10	1
3	13	1
1	15	3
..	..	..
1	17	2, 4, 6, 8, 10
2	18	5, 4, 3, 2, 1

When 1 at 10, 1 at 13, and 3 at 15 give *loss* of 12, which may be made up in *five* ways; and thus an indefinite number of the combinations may be formed.

It should be observed that if the differences opposite the prices *less* than the *mean* are greater together than the sum of the other differences (as in the example), we assign numbers (the *lowest* possible) to the prices less than the mean FIRST, and *vice versa*; *e.g.* of the latter case.

Ex. 3. How much coffee at 25, 24, 23, 22, 21, 19, 18, and 17 cents per pound must be taken to make a mixture worth 20 cents per pound?

Diff's.	20	
3	17	4, 3, 2, 1, 1, 2
2	18	1, 2, 3, 5, 4, 2, etc.
1	19	1, 2, 3, 2, 4 5
..	..	
1	21	1
2	22	1
3	23	1
4	24	1
5	25	1

Here the sum of the differences in excess of the mean is greater than that of the differences below the mean. We therefore assign *first* numbers to the prices which are greater than the mean, viz., 1 at 21, 1 at 22, 1 at 23, 1 at 24, 1 at 25. This gives a *gain* of 15, which may be balanced as above by 1 at 19, 1 at 18, and 4 at 17; or. by 2 at 19, 2 at 18 and 3 at 17, etc., etc.

Ex. 4. A grocer has 12 lb. of brown sugar, worth 10c. per pound, which he wishes to mix with clarified sugar worth 16c. per pound, so that the mixture may be worth 14c. per pound. How many pounds of clarified sugar must he take?

Proceeding as in the previous examples, without reference to the *quantity* of the brown sugar, we find that there must be 1 lb. brown sugar to 2 lb. clarified sugar. But as 12 lb.

of brown sugar are required, we must multiply each of these quantities by 12 in order that the gain and loss may be equal. We shall therefore have 12 × 2 lb. = 24 lb. of clarified sugar.

Ex. 5. A grocer wishes to mix 20 lb. sugar worth 9c. per pound and 10 lb. worth 12c. per pound, with clarified sugar worth 15c., so that the compound may sell at 13c. How much of the clarified must he take?

20 lb. at 9c = $1.80
10 " " 12 = $1.20
30 $3.00

Then, if 30 lb. is worth $3,
1 " " $\$\frac{3}{30}$ = 10c.

The value of 1 " of the mixture is, therefore, worth 10c.

The question may then be read as follows :—

How many pounds of clarified sugar worth 15c. per pound must be mixed with 30 lb. of another kind of sugar, worth 10c. per pound, so that the mixture may be sold for 13c. per pound?

The question in this form has already been fully explained.

Ex. 6. A merchant has West India sugar worth 8c. per pound, and New Orleans sugar worth 13c. He wishes to combine these so as to make a barrel containing 175 lb., which he may sell at 11c. per pound. How many pounds of each kind must he take?

Solving the question without reference to the 175 lb, we find that 2 lb. of West India sugar and 3 lb. of New Orleans sugar will form a mixture worth 11c. per pound. Adding these quantities we find that they form a mixture of 5 lb. But the required mixture is to contain 175 lb., or 35 times 5. We shall therefore have

35 × 2 lb. = 70 lb. West India sugar.
35 × 3 lb. = 105 lb. New Orleans sugar.

Examples X

1. A jeweller melted together 9 oz. of gold 22 carats fine, 12 oz. 18 carats fine, 9 oz. 21 carats fine. What is the fineness of the mixture?

2. If a merchant mixes 7 lb. of sugar worth 8c. a pound with 6 lb. worth 9c., 9 lb. worth 10c. and 10 lb. worth 12c., for how much must he sell the mixture to gain 25%?

3. On a certain day the thermometer ranged at 64° from 6 o'clock to 9, at 76° from 9 to 12, at 85° from 12 to 3, and at 68° from 3 to 6. What was the average temperature?

4. A person mixed 15 gal. of alcohol 80% strong, 12 gal. 90% strong, 23 gal. 60% strong, and 20 gal. 70% strong. What is the strength of the mixture?

5. What quantities of coffee, worth 23c. and 35c., respectively, per pound, must be mixed together so that the compound may be sold for 30c. per pound?

6. What quantity of oats at 35c. per bushel, rye at 60c. per bushel, and barley at 80c., must be taken to form a mixture worth 55c. per bushel?

7. How much tea, worth, respectively, 55c. and 75c. per pound, must be mixed with 20 lb. worth 90c. per pound, in order that the compound may be sold for 70c. per pound?

8. How much water will it require to dilute 60 gal. of alcohol, worth $1.50 per gallon, so that the mixture may be worth only $1.20 per gallon?

9. How many gallons of kerosene oil, worth 60c. per gallon, must be mixed with 12 gal. of coal oil, worth 36c., and 8 gal of Aurora oil, worth 56c., so that the compound may be sold for 50c. per gallon?

10. A farmer has 16 bu. of corn, worth 48c. per bushel, and 12 bu. of oats at 34c. per bushel, which he wishes to mix with rye at 60c. and barley at 80c., in order to sell the

compound at 56c. per bushel. How many bushels of rye and barley will be required?

11. A confectioner mixes three different qualities of candy worth, respectively, 14c., 18c., and 30c. per pound, so as to make a box of 84 lb. How many pounds of each sort must he take so as to sell the compound at an average price of 24c. per pound?

12. A farmer has three different qualities of wool, worth, respectively, 33c., 37c., and 45c. per pound. He wishes to make up a package amounting to 120 lb., which he can afford to sell at 39c. per pound. How many pounds of each kind must he take?

13. How many sheep worth, respectively, $1.50, $2, $2.75, $3, and $4 apiece, can be taken to make a flock of 300 worth $2.50 apiece?

14. How much tea at 40c. and 50c. per pound must be mixed with 36 lb. at 60c., so that the mixture may be sold at 77c., at a gain of 40%?

15. How much tea at 30c., 35c., 40c., 45c., and 50c. must be taken to form a mixture of 100 lb. at 56c., so as to gain $33\frac{1}{3}$%?

16. A farmer wishes to mix corn worth 70c. per bushel with rye worth 75c., barley worth 60c., and oats worth 45c., to make a mixture of 60 bu., which he may sell at 78c. per bushel, at a gain of 20%. How many bushels of each kind must he take?

17. If 16 gal. of spirits at $1.25 per gallon are mixed with 9 gal. at a different price, and 25% is gained by selling a gallon of the mixture at 1.67\frac{1}{2}$, what is the price of the second kind of spirits per gallon?

18. A man paid $165 to 55 laborers, consisting of men, women and boys. To the men he paid $5 per week, to the women $1 per week, and to the boys $\$\frac{1}{2}$ per week. How many were there of each?

19. A man bought calves, sheep, and lambs, 154 in all, for $154. He paid $3½ for each calf, $1⅓ for each sheep, and $½ for each lamb. How many did he buy of each kind?

20. A farmer bought 100 animals for $100. Geese at $½ each, pigs at $3½, and calves at $10. How many animals were there of each kind?

21. A grocer has three kinds of tea; the second being ½ as dear again as the first, and the third ¼ as dear again as the second. If he mixes a certain quantity of the first with twice as much of the second and 112 lb. of the third, and finds the mixture to be ⅓ as dear again as the second sort, of how many pounds does the mixture consist?

ANSWERS TO EXAMPLES IN THE SUPPLEMENT

Examples I. Page 144

1. 16.   2. 32.   3. 42.   4. 79.   5. 85.   6. 64.
7. 34.   8. 73.   9. 92.   10. 99.   11. 39.   12. 63.

Examples II. Page 145

1. 245.   2. 531.   3. 307.   4. 670.   5. 128
6. 179.   7. 463.   8. 103.   9. 256.   10. 579.
11. 438.   12. 507.   13. 686.   14. 708.   15. 888.
16. 512.   17. 4968.   18. 8765.

Examples III. Page 146

1. .73.   2. .364.   3. 30.02.   4. $\frac{11}{12}$.   5. $\frac{5}{7}$.
6. $1\frac{5}{7}$.   7. $7\frac{3}{8}$.   8. 1.709.   9. 8.320.   10. .495.
11. 2.516.   12. .822.   13. .908.   14. .693.   15. 1.966.
16. 1.473.

Examples IV. Page 147

1. 27.   2. 45.   3. 6.3.   4. 13.   5. 54.   6. 8.1.

Examples V. Page 147

1. 879.   2. 3420.   3. 93, 31, 62.
4. 55, 110, 220, 440.   5. 35856144, 35832196.   7. 1512.   8. 9.

EXAMPLES VI. PAGE 149

1. 2.2894. 2. 3.1072. 3. 6.0822. 4. 7.2112. 5. .9654.
6. .4308. 7. 2.1006. 8. .8372. 9. 2.0614.

EXAMPLES VII. PAGE 153

1. (*a*) $4600; (*b*) $70; (*c*) $562.50; (*d*) $1050; (*e*) £537 10*s*
2. (*a*) $36; (*b*) $150.80; (*c*) $137.50; (*d*) £152 1*s*. 3*d*.; (*e*) £7 10*s*. 3*d*.
3. $9030.
4. $4.40.
5. 5%.
6. 1105.
7. 45\frac{5}{8}$, 13\frac{2}{21}$
8. 80 to 83; $3½.
9. 16\frac{11}{13}$.
10. $520; 6%.

EXAMPLES VIII. PAGE 155

1. 7½ mo.
2. 4$\frac{1}{12}$ mo.
3. March 18.
4. 8⅓ mo. after Nov. 6.
5. 4 mo. after 14 mo.
6. 12⅓ mo.
7. 13 mo.
8. 5 mo.
9. 3½ mo.
10. 7½ mo.

EXAMPLES IX. PAGE 158

1. $431; May 9. 2. $190; March 11. 3. $395; May 28.

EXAMPLES X. PAGE 164

1. 18.825 carats. 2. 12½c. 3. 73.25°. 4. 72$\frac{4}{7}$%.
5. 5 lb. of first, 7 lb. of second.
6. 30 bu. oats; 20 bu. rye; 20 bu. barley.
7. 50 lb. at 55c.; 30 lb. at 75c.
8. 15 gal. water.
9. 12 gal. kerosene.
10. 14 bu. rye; 14 bu. barley.
11. 18 lb. at 14c.; 18 lb. at 18c.; 48 lb. at 30c.
12. 36 lb. at 33c.; 36 lb. at 37c.; 48 lb. at 45c.
13. 60 at $1.50, 60 at $2, 120 at $2.75, 30 at $3, and 30 at $4.
14. 9 lb. at 40c., and 9 lb. at 50c.
15. 10 lb. at 30c., 10 lb. at 35c., 20 lb. at 40c., 50 lb. at 45c. and 10 lb. at 50c.
16. 12 bu. corn, 24 bu. rye, 12 bu. barley, 12 bu. oats.
17. $1.50.
18. 30 men, 5 women, 20 boys.
19. 14 calves, 42 sheep, 98 lambs.
20. 92 geese, 4 pigs, and 4 calves.
21. 130 lb.